INITIAL MATHEMATICS

A Guidebook for Parents and Teachers

mirambika
Free Progress School
Research Centre for Integral Education and Human Values
Sri Aurobindo Education Society

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Thank you, mirambika, for setting us on the path of free progress, with your open environment, and a pace gentle enough to explore mathematics learning in new ways.

Thank you, Srinath bhaiya, a teacher trainee in mirambika since 2005, who went on to spend some years as a diya here before moving on to work with Auromira Vidyamandir in Kechla, Koraput, Odisha. Srinath's interest in mathematics drew him close to Jasbir bhaiya's outlook towards the teaching of mathematics to young children. Greatly inspired by him, Srinath tried out many experiments with the children and eventually set up the Mathematics Lab in mirambika. He initiated the process of writing this book. Without his sincerity and dedication, this book may never have been.

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Thank you to Nature - to mother earth, all the animals, insects, leaves, fruits, flowers - with which we have spent so many magical hours, lost in numbers and patterns.

Most of all thank you children, for being the reason for this book. This book is dedicated to all the children of the world - eyes shining with the wonder of the universe, of which mathematics is such a vital part.

Most importantly, our profound gratitude goes to Jasbir Bhaiya, for planting the seed of Initial Mathematics; for helping us see in innovative ways; for guiding fellow teachers and parents to look at mathematics not just functionally but as a field of elegance and beauty; and for practising all these for 18 indefatigable years with the children and diyas of mirambika.

mirambika family

Jasbir Bhaiya as we know him...

Jasbir Singh Malik

It is morning hours in mirambika Free Progress School, at the time of the decade beginning 2001. School is in session. At about 10am, the little children of the ages of 8 to 9 dash out of their classroom suddenly and in excitement, screaming "Jasbir Bhaiya! Jasbir Bhaiya!" and pounce on a tall, elderly, bespectacled gentleman near the sandpit, who lights up into a wide grin, slows down his gait and engages in a friendly banter with the children as he hails, "Jai Pakistan!...Pakistan will win!" (referring to past or impending cricket matches), much to the children's dismay, who immediately protest with, "Jai Bharat!" weighing upon him, clinging onto both his arms, and with two or three tiny arms around his waist, walk him to the portals of the classroom. He was nothing but a wonderful sight of joy, laughter and benevolence in the midst of the children.

Then in another scenario, a strong voice with a clipped British accent breaks through the natural quietness of mirambika Free Progress School, telling the story of a bus with passengers getting on and off in the course of a journey and asking how many were left on the bus at the end of the journey. In hushed silence the children, all of 10 years of age, raise their hands and whisper the answer to bhaiya.

Jasbir Singh Malik, or Jasbir bhaiya, lovingly hailed as the Yoda of our times, joined mirambika in 1989 in response to an advertisement seeking a multi-faceted volunteer teacher. His multifaceted talents, acting being one of them, found him not only taking mathematic classes but also teaching English, and exploring literature through Shakespeare, theatre and poetry recitation. He himself had taken up some roles in films. His coveted voice and diction made him a natural choice for voice-overs for our in-house productions such as films on mirambika. Wide in his outlook, he had a complete view of the world. He had a keen interest in politics and the living conditions of his fellow countrymen. He generously shared his views on how to run a school so that the child benefits truly. His advise on how to manage and deal with parents was invaluable. He believed in the alumni and was always keen on bringing them together in order to keep in touch with the younger generation. A man of conviction, he inspired many more to join as volunteers in mirambika, on a journey of discovery. A stickler for punctuality, he arrived on time for all his classes in an autorickshaw. He practiced this virtue till his last days in mirambika, rain, shine or cold.

He stayed on with us till 2014, all the while opening to mirambika's ways like a sunflower to the sun, and chiselled a pathbreaking approach to the teaching and learning of mathematics in the younger age groups. Taking up a comprehensive view of the teaching of mathematics, he emphasised first of all on our inner attitude towards numbers. For him, mathematics was an integral part of existence. He was particularly insistent that parents and facilitators also learn this wholesome approach to numbers. He often conducted classes for both parents and diyas too. Thus he became a mentor for the whole school. Initial Mathematics, as he pointed out, relied on establishing a personal relationship with numbers, and their quantification, patterns and designs, preparing children to deal with abstract mathematical language at a later stage. All his efforts were towards garnering the children's interest and their joyful engagement with numbers in everyday life. His usual conversations with children were candid, full of wit and sprinkled with generous doses of a sense of humour which children took to naturally. Even in those conversations, Jasbir bhaiya injected loads of well thought out questions and prompts that would stimulate the young minds to intuitively elicit a series of responses approaching a quantification.

Initial Mathematics was a term that Jasbir bhhaiya was instrumental in coining. He initiated mental math and oral math calculations. He once told a senior diya that he used mathematics as a tool to develop the faculty of listening, concentration, and intuitive thinking amongst the children. Intuition is the key word of Initial Mathematics. He emphasised on addressing and observing the intuitive faculty of the children. He never wrote off an answer as wrong but instead, wrote all the answers on the board and urged the children to reconsider the working out of the given problem. According to a senior diya, children enjoyed his style of teaching and were not afraid of making mistakes. Working alongside him was indeed a valuable lesson in life.

Jasbir bhaiya initially facilitated learning of mathematics in older groups of children. However, for a significant number of years he started taking up Initial Mathematics for young children in the 9-10 age bracket, with a keen eye on how diyas handled working with numbers in the very young, from the age of 4-plus to 8. He was very sensitive to children's needs. According to one diya who was hand-held by him, he insisted on the intuitive response to numbers and would often urge her to, "… think with your heart!" Above all, his innate love flowed ceaselessly towards the children who sensed it. Children loved him. They did not take his affection for granted. His work would all be submitted on time. Having had his father serving in the army, discipline came naturally to him and he religiously expected it of the children.

Jasbir Bhaiya did not confine himself to the teaching of mathematics alone. He taught children and adults around him values of life through example. There was one incident where he was having a discussion with 9-10 year olds where children expressed their aversion to cleaning the toilet which only servants in their homes did. In the course of that particular discussion, Jasbir bhaiya got up, marched to a toilet at the back of the school and cleaned the toilet with his bare hands, with the children watching.

Between 2013 and 2014, a more senior and frail Jasbir bhaiya returned to mirambika after some years of absence and took up very seriously the facilitation of diyas anchoring younger groups Initial Mathematics. He was embarking on a project of documenting Initial Mathematics. He took up many learning sessions with these diyas, including one to one sessions, developing a sense of how to work with young children through numbers and gradually and with patience, love and care, consolidated the Initial Math approach in mirambika. It was during this time that he began to write the paper on Initial Mathematics, making comparisons with the traditional way of mathematics teaching. Jasbir bhaiya left us without completing that paper. This is what we publish in this book as Chapter 3, in two parts, Part 1 and 2. We leave these parts largely unedited, just as Jasbir Bhaiya had left them, to preserve his unique thoughts on numbers and how children relate with them and how their number sense can be facilitated sensitively and intuitively. Though incomplete, one can meet the completeness of his ideas on Initial Mathematics here as he keenly championed the interest of little children facing the world of numbers as they are and laid some basic guidelines on how they can be naturally, seamlessly and intuitively eased into understanding it in their own way, expressing it in their own language. More than constructing a mathematics curriculum, Jasbir bhaiya has provided guidelines for the attitude that a teacher must hold towards the children and towards numbers, emphasising on the psychology of the child and his or her readiness to engage with the concept of numbers.

His approach to teaching Initial Mathematics was indeed path breaking. He was a member of a committee on Primary Mathematics and together with Pravin Sinclair was able to create a course which is even offered today by IGNOU to all in the teaching of Primary Mathematics.

This Initial Mathematics project was indeed his loving gift to the world of children.

"The finest present one can give to a child would be to teach him to know himself and to master himself"

The Mother

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INTRODUCTION

Does an artist teach by following a textbook?

An art teacher does with children what is natural to the subject – painting. He goes from child to child making suggestions and providing guidance as the child instinctively scrawls or details the lines and splashes or spreads the colours to his or her heart's call.

In much the same way, teaching, or rather, facilitation of learning must be creative while encouraging the children to explore, experience and find their own way of creating and solving the problems of their life rather than pursue the dictate of a textbook. It is best to await the emergence of a natural curiosity in children to know and learn. Perhaps giving space, time and stimulus to this emergence may be our sole work as facilitators of learning.

There is a great deal that can be done to prepare for the adventure that mathematics offers; playing games such as chess, bingo, snakes and ladders, solving puzzles, or simply making up a game, exposing children to different situations in problem solving, observing the patterns in nature and in one's inner and outer life are but a few examples of the facilitation role teachers can take up. The goal before us, in the dealing with mathematics is to nurture their intuitive capacity, and to help children become active and creative mathematical thinkers. Accordingly, we need to make the facilitation of the learning of mathematics welcoming, creative, experiential and most importantly, joyful.

Considerable research has been undertaken over the past 40 years on the overall growth of the children at mirambika, a free-progress school, practicing The Mother and Sri Aurobindo's philosophy in education. The ideal school is a place where children do not feel pressurised with textbook learning, remembering formulae or doing homework. It is a school where the innate natural curiosity is kindled and learning takes place spontaneously and joyously.

This book holds the key to how one could view numbers as an integral, essential feature of our lives, as well as a treasury of ideas on how numbers could be brought alive in the child's consciousness through collaborative activities in an environment of acceptance and encouragement and while at play. Our aspiration is to put a suggestion in the present moment that can ignite a certain way of approaching numbers with children so that generations may embrace mathematics in all its richness. It is important to start this process with the foundations. This book holds a chest of experiences gleaned from 40 years of working with children at the primary level on numbers.

Readers may come across the term 'diya' in the book from time to time. It is an endearing term addressing the facilitators of Mirambika. Combining the first syllable of 'didi' or sister in Hindi, which is 'di' and the ending syllable of 'bhaiya', which is 'ya', derives diya. Diya also means the lamp, which gives the term another significance and serves as a constant reminder to the diyas of the high place they hold in guiding the learners in the path of light.

We start the book with a brief idea about the evolution of mathematics and its need and application in different fields over time across human civilisations starting with the primitive man leading to the evolution of numerical names and symbols, decimal systems, its growth through trade and commerce, and its importance in astronomy. Included is a section on the greatest contribution of India in the discovery of positional or the place value system and its widespread use now as the universal alphabet. We put across the succinct suggestion that mathematics is possible without learning any formal math with the implication that perhaps the knowledge of mathematics as intuitive. We conclude this chapter pondering over the necessity of learning mathematics for the future of the human race.

The second chapter delves into the importance of mathematics for the integral growth of a person, bringing out latent qualities and faculties. The Integral Philosophy, as propounded by Sri Aurobindo and as practiced by The Mother in Integral Education is discussed at length concluding with the proposition that the subtler psychic qualities like inner joy, love and intuitive understanding can be developed and enhanced through mathematics.

The third chapter holds views on Initial Mathematics by its proponent, Jasbir Singh Malik in three parts. The writings featured in the first two parts of chapter three were never completed by the author. We present his writings as we received them, with minor editions, in order to preserve his fine thoughts on mathematics learning in very young children gained through long years of experience coupled with a passion for numbers and a keen mind contemplating on how children related with numbers intuitively. The first part is the description of a needed re-orientation in the adult views on how a child should learn mathematics with a brief comparison on approaches to initial mathematics from a child and the significant adults in her life – teachers and parents. The second part gives a good airing to the role of intuition in a child's life, the influence of sensory experiences and the environment in the child's life and how traditional approaches pose an opposite current to the natural flow of a child's experiences and intuition. Ironically, in the absence of adult intervention, there appears no confusion between reason and intuition in the child. In this instance, the learning of numbers is akin to acquiring a language by listening and observing. It only calls for facilitation by the teacher instead of instructions. This part of the chapter also holds an array of examples of basic mathematical functions for the practitioner to try out in the classroom with children. The third part of this chapter presents the transcripts of an interview with the author of this chapter and the inspiration behind this book.

Chapter four introduces the preliminary concepts that should be ideally brought within the fold of children's experiences through the play-way method. Pre-number aspects of mathematical dealings, such as classification, grouping, matching, ordering, seriation, and pairing with one to one correspondence of concrete objects as well as relevant activities are described. Pattern design done by children has taken an important place in this chapter.

Chapter five examines counting and estimation on various levels and experiences with suggested activities and games while chapter six explains the innovative use of abacus as a part of the mathematical process.

In chapters seven and eight, an endeavour is made to build up an understanding of the basic operations of addition, subtraction, multiplication and division using daily life experiences and concrete materials offered by nature as well as card games. This paves the way for the deliberate practice of mental mathematics.

Chapter nine deals with simple concepts of fraction using shapes and paper. The numerator and denominator are introduced to the children through clearly set out games. The use of the four operations in fraction are explored through various examples.

Chapter ten progresses into a thorough overview of word problems through real life situations. The identification by the child of key words is explored. Many examples of word problems are presented.

The final chapter introduces projects taken up in mirambika with young children in different age groups, such as those on, Pattern and design, Time, Coin, Measurement and Market. This chapter gives some practical ideas on how numbers lend themselves naturally to certain projects and these experiences can bring alive the basic principles of mathematics to children spontaneously, albeit with careful planning by the facilitators of the project (i.e. teachers).

The appendix details a tentative Initial Mathematics curriculum in mirambika from age groups 3+ to 8+. The bibliography completes the book.

It is our hope that this book will be of help to practitioners as well as parents seeking to build a generation that will embark on an adventurous journey of discovering the wonders of numbers through mathematics, a process we believe is as natural as learning a language, and in this instance, the beautiful language of numbers.

The mirambika family
May 2021

CHAPTER 01

The Story of Mathematics

The evolution of Mathematics has not happened accidentally. It has evolved according to the needs of developing civilisations.

Archaeologists, linguists, anthropologists and others studying early societies have found that number ideas evolve slowly. There will typically be a different word or symbol for two people, two birds, or two stones. Only slowly does the idea of '*two*' become independent from the things that there are two of. It is the same, of course, for other numbers. In fact, specific numbers beyond three are unknown in some less developed languages.

We can see that the use of mathematics was widespread in different areas of the social, cultural and physical environment. People used to solve different life-related problems using mathematics. Therefore, a lot of development took place before mathematics came to be written.

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The Mayan, the Chinese, the Civilisation of the Indus Valley, the Egyptians, and the region of Mesopotamia between the Tigris and Euphrates rivers all had developed impressive bodies of mathematical knowledge by the dawn of their written histories. In each case, what we know of their mathematics comes from a combination of archaeology, the references of later writers, and their own written record.

Mathematical documents from Ancient Egypt date back to 1900 B.C. There was a practical need to redraw field boundaries after the annual flooding of the Nile. This, along with the fact that there was a small leisure class with time to think, helped to create task-oriented, practical mathematics. A base-ten numeration system was able to handle positive whole numbers and some fractions. Algebra was developed only far enough to solve linear equations and, of course, calculate the volume of a pyramid. It is thought that only special cases of The Pythagorean Theorem were known (ropes knotted in the ratio 3:4:5 may have been used to construct right angles).

What we know of the mathematics of Mesopotamia comes from cuneiform writing on clay tablets which date back as far as 2100 B.C. Sixty was the number system base — a system that we have inherited and preserve to this day in our measurement of time and angles. Among the clay tablets are found multiplication tables, tables of reciprocals, squares and square roots. A general method for solving quadratic equations was available, and a few equations of higher degree could be handled. From what we can see today, both the Egyptians and the Mesopotamians (or Babylonians) stuck to specific practical problems; the idea of stating and proving general theorems did not seem to arise in either civilisation.

The Classic Mayan civilisation (250 BC to 900 AD) also developed the zero and used it as a placeholder in a base-twenty numeration system. Astronomy played a central role in their religion and motivated them to develop mathematics. It is noteworthy that the Mayan calendar was more accurate than the European one at the time the Spanish landed in The Yukatan Peninsula.

Looking back at a more primitive time, early hunters, while directing a projectile or an arrow, were instinctively using their mathematical minds to compute angles and directions.

As primitive people began to live in a group (as a family, in a village, in a community) their needs and requirements underwent a change. They started cultivation, and keeping different domesticated animals like sheep and cows. Gradually the number of materials and livestock increased, and it was difficult to keep track of everything.

How would the shepherd or cowherd know that none of the sheep or cows was missing? This is where natural human ingenuity must have filled the need. To cope with larger number of objects, it is probable that ancient humans resorted to some grouping (such as we group in tens, hundreds, thousands and so on). Before our decimal number system there were many ways of grouping the number throughout the world. However, we commonly find some multiples of five (five, ten, fifteen...) being used as the basis of grouping (what is known as tally marks for recording our observations). This is due to the fact that primitive humans, like children, probably used their fingers to check their count.

Numbers in every early societies were typically represented by groups of line.

In all early civilisations, the first expression of mathematical understanding appears in the form of counting systems. Numbers in very early societies were typically represented by groups of lines, though later different numbers came to be assigned specific numeral names and symbols (as in India) or were designated by alphabetic letters (such as in Rome). Although we take our decimal system for granted, not all ancient civilisations based their numbers on a ten-base system. As we have seen earlier, in ancient Babylon, a sexagesimal (base 60) system was in use.

The Decimal System in Harappa

In India a decimal system was already in place during the Harappan period, as indicated by an analysis of Harappan weights and measures. Weights corresponding to ratios of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500 have been identified, as have scales with decimal divisions. A particularly notable characteristic of Harappan weights and measures is their remarkable accuracy. A bronze rod marked in units of 0.367 inches points to the degree of precision demanded in those times. Such scales were particularly important in ensuring proper implementation of town planning rules that required roads of fixed widths to run at right angles to each other, for drains to be constructed to precise measurements, and for homes to be constructed according to specified guidelines. The existence of a gradated system of accurately marked weights points to the development of trade and commerce in Harappan society.

The Indian Numeral System

Although the Chinese were also using a decimal based counting system, the Chinese lacked a formal notational system that had the abstraction and elegance of the Indian notational system, and it was the Indian notational system that reached the Western world through the Arabs and has now been accepted as universal. Several factors contributed to this development whose significance is perhaps best stated by French mathematician, Laplace: "*The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions.*"

Indian notational system that reached the Western world through the Arabs, brilliant as it was, this invention was no accident.

Brilliant as it was, this invention was no accident. In the Western world, the cumbersome Roman numeral system posed a major obstacle, and in China the pictorial script was a hindrance, but in India, almost everything was in place to favour such a development. There was already a long and established history in the use of decimal numbers. Philosophical and cosmological constructs encouraged a creative and expansive approach to number theory. *Panini's* studies in linguistic theory and formal language, the powerful role of symbolism and representational abstraction in art and architecture may have also provided an impetus. These must have been reinforced by the rationalist doctrines and exacting epistemology of the *Nyaya Sutras*, and the innovative abstractions of the *Syadavada* and Buddhist schools.

Virtually every Indian state produced great mathematicians

Influence of Trade and Commerce, Importance of Astronomy

The growth of trade and commerce, particularly lending and borrowing demanded an understanding of both simple and compound interest, which probably stimulated interest in arithmetic and geometric series. *Brahmagupta's* description of negative numbers as debts and positive numbers as fortunes points to a link between trade and mathematical study. Knowledge of Astronomy — particularly knowledge of the tides and the stars — was of great import to trading communities who crossed oceans or deserts at night. This is borne out by numerous references in the *Jataka* tales and several other folk-tales.

The young person who wished to embark on a commercial venture was inevitably required to first gain some grounding in astronomy. This led to a proliferation of teachers of astronomy, who in turn received training at universities such as at Kusumpura (Bihar), Ujjain (Central India) or smaller local colleges or *Gurukuls*. This also led to the exchange of texts on astronomy and mathematics amongst scholars, and the transmission of knowledge from one part of India to another. Virtually every Indian state produced great mathematicians who wrote commentaries on the works of other mathematicians (who may have lived and worked in a different part of India many centuries earlier). Sanskrit served as the common medium of scientific communication.

Significance of the decimal number system

"Finally it all came to pass as though across the ages and the civilisations, the human mind had tried all the possible solutions to the problem of writing numbers, before universally adopting the one which seemed the most abstract, the most perfected and the most effective of all." In these memorable words, the French-Moroccan scholar Georges Ifrah, the author of the monumental The Universal History of Numbers, sums up the many false starts by many civilisations until the Indians hit upon a method of doing arithmetic which surpassed and supplanted all others—one without which science, technology and everything else that we take for granted would be impossible. This was the positional or the place value number system. It is without a doubt the greatest mathematical discovery ever made, and arguably India's greatest contribution to civilisation.

The term 'Arabic numerals' is a misnomer; the Arabs always called them 'Hindse' numerals.

The term 'Arabic numerals' is a misnomer; the Arabs always called them 'Hindse' numerals. What is remarkable is the relatively unimportant role played by the Greeks. They were poor at arithmetic and came nowhere near matching the Indians. Babylonians a thousand years before them were more creative, and the Maya of pre-Colombian America far surpassed them in both computation and astronomy. The Greek Miracle is a modern European fantasy.

The discovery of the positional number system is a defining event in history, like man's discovery of fire. It changed the terms of human existence. While the invention of writing by several civilisations was also of momentous consequence, no writing system ever attained the universality and the perfection of the positional number system. Today, in the age of computers and the information revolution, computer code has all but replaced writing and even pictures. This would be impossible without the Indian number system, which is now virtually the universal alphabet as well.

What makes the positional system perfect is the synthesis of three simple yet profound ideas: zero as a numerical symbol; zero having 'nothing' as its value; and the zero as a position in a number string. Other civilisations, including the Babylonian and the Maya, discovered one or another feature but failed to achieve the grand synthesis that gave us the modern system. Of the world's civilisations, the Mayas came closest. They, like the Babylonians, had an idea of the zero, but never learnt how to operate with it.

In Ifrah's words: "*The measure of genius of the Indian civilisation, to which we owe our modern system, is all the greater in that it was the only one in all history to have achieved this triumph.*" Modern civilisation rests on the modern number system. The decimal system is just a special case of it.

Throughout history, whenever there was an important problem, a revolution paved the way, and a solution emerged. Today, through a series of little and large revolutions, mathematics has a special place. It has become the root of our lives, its necessity felt at each and every step.

Yet, what of the many who are touched neither by teacher nor school? How then are they able to maintain their lives? This line of enquiry has led to questions like, "Is mathematics required only for counting, keeping record, calculating the expenditure of daily life, managing the office, managing the company? Or do we use it to discover new things to make our lives easier?" A street side peanut seller makes his life easier without going to any formal school and without learning any formal maths. From where does he acquire knowledge of mathematics to make his life easier? If we consider this deeply, it follows the knowledge is within, simply waiting to come out. This inner knowledge blossoms according to the environment provided. The skills of mathematics develop instinctively, whenever necessary.

Having considered the past and the present, we are left with another question, one that gazes into tomorrow.

Is learning mathematics necessary for the future of the human race?

…never come to me saying, "I am no good at this subject, I shall never understand philosophy" or "I shall never be able to do mathematics" or... It is ignorance, it is sheer ignorance. There is nothing you cannot understand if you give your brain the time to widen and perfect itself. And you can pass from one mental construction to another: this corresponds to studies; from one subject to another: and each subject of study means a language; from one language to another, and build up one thing after another within you, and contain all that and many more things yet, very harmoniously, if you do this with care and take your time over it. For each one of these branches of knowledge corresponds to an inner formation, and you can multiply these formations indefinitely if you give the necessary time and care.

The Mother

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CHAPTER 02

Integral Education

In this chapter the focus is on the role of mathematics in Integral Education or how mathematics help to develop the latent faculties in a human being.

Mathematics has a distinctive place in integral education led by The Mother and Sri Aurobindo. Let us try and understand what they have said about Integral Education, and mathematics.

An education which takes into account the entire complexity of man's nature can rightly be termed an "integral education".

Central aim is the building of the powers of the human mind and spirit............

A human being is constituted of many parts. Body (the physical being) is the outermost expression of the innermost reality (the psychic being). In between are layers of the emotional nature (the vital being) and the thinking individuality (the mental being). An integral education naturally takes into account the growth and perfection of each of these parts, integrating the entire being around that which is the highest, deepest and widest in each human being.

As the Mother says, "Education to be complete must have five principal aspects corresponding to the five principal activities of the human being: the physical, the vital, the mental, the psychic and the spiritual. Usually, these phases of education follow chronologically the growth of the individual; this, however, does not mean that one of them should replace another, but that all must continue, completing one another until the end of his life."[1]

The Mother has explained about the goals to be achieved in these words,
"the body will be the expression of a perfect beauty and harmony"
"the vital will manifest an invincible power and strength"
"the mind will be the vehicle of infallible knowledge"
"the psychic will be the vehicle of true and pure love"[2]

Psychic Education

As the Mother says, "The psychic being is the representative of the Divine in the human being."[3] To explain it further she says, "Every human being carries hidden within him the possibility of a greater consciousness which goes beyond the bounds of his present life and enables him to share in a higher and a vaster life. What the human mental consciousness does not know and cannot do, this consciousness knows and does. It is like a light that shines at the centre of the being, radiating through the thick coverings of the external consciousness. Some have a vague intimation of its presence; a good many children are under its influence, which shows itself very distinctly at times in their spontaneous actions and even in their words."[4]

The discovery of the soul, the real man within, is truly the first great goal of human life. Education can and should move in this direction by helping "the child to educate himself, to develop his own intellectual, moral, aesthetic and practical capacities and to grow freely as an organic being"[5]

To become aware of the psychic consciousness is the most important aspect of integral education. The main focus should be to bring the psychic to the forefront to guide our external consciousness; physical, vital and mental being.

The psychic being is the representative of the Divine in the human being.

Physical Education

According to Sri Aurobindo, "If our seeking is for a total perfection of the being, the physical part of it cannot be left aside; for the body is the material basis, the body is the instrument which we have to use."[6]

"Physical education has three principal aspects:

1. control and discipline of the functioning of the body,
2. an integral, methodical and harmonious development of all the parts and movements of the body and
3. correction of any defects and deformities."[7]

The Mother says to achieve this we have to go through the discipline - tapasya of beauty. "Its basic programme will be to build a body that is beautiful in form, harmonious in posture, supple and agile in its movements, powerful in its activities and robust in its health and organic functioning. To achieve these results, it will be good, as a general rule, to make use of habit as a help in organising one's material life, for the body functions more easily within the framework of a regular routine." [8]

Perfection of the physical body to make it into a fit instrument for a perfect life must be the aim of an efficient physical education.

To make the physical being active and energetic, the role of vital being is very much essential, since vital being is the seat of energy, power and life force. Without it no work can be done.

Vital Education

The vital is a vast kingdom, full of forces acting and reacting upon one another, the very nexus of man's life and the motive power of his action - for good or for evil. The organization and training of this complex of forces is of the utmost importance for the building up of character.

To proceed further in this direction Sri Aurobindo says, "the vital is the Life-nature made up of desires, sensations feelings, passions, energies of action, will of desire, reaction of the desire-soul in man and of all that play of possessive and other related instincts, anger, fear, greed, lust, etc., … it is the vital which holds power, energy, enthusiasm, effective dynamism."[9]

The Mother has said that the training of the vital is the most important, the most indispensable of all forms of education. She has described it in a very scientific way.

".... the training of the vital is the most important, the most indispensable of all forms of education.

"…vital education has two principal aspects, very different in their aims and methods, but both equally important. The first concerns the development and use of the sense organs. The second the progressing awareness and control of the character, culminating in its transformation."[10]

"By the education of the senses the growth of one's general education is aided. if one learns to see well, exactly, precisely, hear well, touch to know the nature of things, smell to distinguish between different odours, taste well - all these are a powerful means of education."[11]

"To this general education of the senses and their functioning there will be added, as early as possible, the cultivation of discrimination and of the aesthetic sense, the capacity to choose and adopt what is beautiful and harmonious, simple, healthy and pure."[12]

And this quite naturally lead to the second aspect of vital education which concerns the character and its transformation. "To become conscious of the various movements in oneself and be aware of what one does and why one does it, is the indispensable starting-point."[13]

"To sum up: one must gain a full knowledge of one's character and then acquire control over one's movements in order to achieve perfect mastery and the transformation of all the elements that have to be transformed."[14]

To make the vital a docile instrument, the role of mind is very important. Because the mind with its reasoning power can control the lower nature of the vital and direct it in a constructive way.

Mental Education

The Mother says, "A true mental education, which will prepare man for a higher life, has five principal phases. Normally these phases follow one after another, but in exceptional individuals they may alternate or even proceed simultaneously. These five phases, in brief, are:

1. Development of the power of concentration, the capacity of attention.
2. Development of the capacities of expansion, widening, complexity and richness.
3. Organisation of one's ideas around a central idea, a higher ideal or a supremely luminous idea that will serve as a guide in life.
4. Thought-control, rejection of undesirable thoughts, to become able to think only what one wants and when one wants.
5. Development of mental silence, perfect calm and a more and more total receptivity to inspirations coming from the higher regions of the being."[15]

Sri Aurobindo has pointed out to the nature of current education which is largely off balance in the nurturing of the mental faculty, let alone the whole being. He says, "The first fundamental mistake has been, therefore, to confine ourselves to the training of the storing faculty, memory and the storage of facts and to neglect the training of the three great manipulating faculties, viz. the power of reasoning, the power of comparison and differentiation and the power of expression."[16]

According to him, concentration and observation is the first quality of the mind that has to be developed. The other faculties like memory, comparison, analogy, reasoning and judgment should be trained simultaneously. Apart from these, imagination is one of the important instruments which needs to be nurtured in the same manner.

The intellect, or buddhi, is the real instrument of thought and that which orders and organizes the knowledge acquired by the other parts of the mental machine. Sri Aurobindo conveniently distinguishes this as the most important organ for the educationist. He says,"Intellect is an organ composed of several groups of functions, divisible into two important classes, the functions and faculties of the right-hand, the functions and faculties of the left-hand. The faculties of the right-hand are comprehensive, creative and synthetic; the faculties of the left-hand critical and analytic. To the right hand belong judgment, imagination, memory, observation to the left-hand comparison and reasoning. The critical faculties distinguish, compare, classify, generalise, deduce, infer, conclude; they are the component parts of the logical reason. The right hand faculties comprehend, command, judge in their own right, grasp, hold and manipulate. The right hand mind is the master of knowledge, the left hand its servant. The left-hand touches only the body of knowledge, the right hand penetrates its soul. The left hand limits itself to ascertained truth, the right-hand grasps that which is still elusive or unascertained. Both are essential for the completeness of the human reason. These important functions of the machine have all to be raised to their highest and finest working-power, if the education of the child is not to be imperfect and one-sided."[17]

....concentration and observation is the first quality of the mind that has to be developed.

Sri Aurobindo adds that there is another layer of faculty which, "... not as yet entirely developed in man, is attaining gradually to a wider development and more perfect evolution. The powers peculiar to this highest stratum of knowledge are chiefly known to us from the phenomena of genius, sovereign discernment, intuitive perception of truth, plenary inspiration of speech, direct vision of knowledge to an extent often amounting to revelation, making a man a prophet of truth. These powers are rare in their higher development, though many possess them imperfectly or by flashes." [18]

Sri Aurobindo recognises this as the element of genius in the pupil and he says humanity could have not advanced to its present stage without it and the perfect development of this element of genius.

The Mother and Sri Aurobindo on "Mathematics":

Quest for knowledge is inherent in every human being. It is a never-ending process. If we observe carefully a child doing something, we will be able to notice how his mind is occupied with a desire to learn, to know, to understand the world around and to master the skills. It finds a joy in developing its innate capacities and faculties.

Encouraging the children of the ashram, The Mother says, "There are a lot of things that we need to know, not because we find them specially interesting but because they are useful and even indispensable; mathematics is one of them."[19]

Sri Aurobindo has also pointed out that mathematics is one of the important aspects of developing one's knowledge. To quote him, "The humanities, mathematics and science are therefore the three sisters in the family of knowledge"[20]

If we look at Indian civilisation, we will find that mathematics today owes a huge debt to the outstanding contributions made by the mathematicians of India over many hundreds of years. In the words of Sri Aurobindo, "Not only was India in the first rank in mathematics, astronomy, chemistry, medicine, surgery, all the branches of physical knowledge which were practised in ancient times, but she was, along with the Greeks, the teacher of the Arabs from whom Europe recovered the lost habit of scientific enquiry and got the basis from which modern science started. In many directions India had the priority of discovery, — to take only two striking examples among a multitude, the decimal notation in mathematics or the perception that the earth is a moving body in astronomy, *cala prthvı sthira bhati* the earth moves and only appears to be still, said the Indian astronomer many centuries before Galileo."[21]

It is a fact that, mathematics evolved as a science which deals with the quantitative aspects of our life and knowledge. Today we have reached a remarkable position in science with the help of mathematical language. Mathematics helps science to express its theses and outcomes better than other languages.

The Mother says, "Numbers are ways of speaking. It is a language, as all the sciences, all the arts, everything that man produces; it is always a way of speaking. It is a language. If one adopts this language it becomes livings, expressive, useful."[22]

Every different branch of mathematics, like geometry, arithmetic etc. have their own set of languages. The Mother writes about arithmetic as follows: "Arithmetic is also a science of order. Even a very small child takes delight in repeating numbers in the right order. He soon discovers that there is no meaning in saying: one, five, three, ten, two, as he counts his fingers or his marbles. He counts: one, two, three, four; and all mathematics comes from that."[23]

Mathematics helps to develop the logical faculty and makes the mind supple, clear and accurate in its working. She adds, "As for arithmetic, I am much more in favour of practical than of written arithmetic, with an emphasis on the development of the faculty of mental arithmetic. It is more difficult, but it greatly increases the capacity for inner visualisation and reasoning. It is a very effective way of developing true intelligence instead of memorised knowledge.

When one knows mental arithmetic and understands arithmetic, it then takes very little time to learn written arithmetic.

With the help of similar objects — you can begin with the children themselves for small numbers and then take pebbles and counters when it comes to tens and hundreds."[24] About multiplication table The Mother writes …"you are made to learn by heart the multiplication tables; if you constantly use them, you will remember them, but if by chance for years you remain without using them, you will forget them completely. But if you understand the principle, you will be able to remember them. You see, the principle of multiplication, if you understand it with a mathematical sense, you will no longer need to learn it by heart, the operation will be done quite naturally in your brain…"[25]

The role of Mathematics in integral growth:

Most of us agree that teaching mathematics addresses the growth of mental faculties, but there is much more to it. Along with mental faculties it also helps in the growth of the psychic, vital and physical aspects of one's life. That is why the mind trained through the study of mathematics is more capable of leading a disciplined life. Mathematics helps to bring forth many qualities and habits.

The study of mathematics helps us to develop our mental faculties like logical thinking, reasoning, analysis and synthesis, organizing, imagination, observation, creativity, concentration and decision-making. The step-by-step procedure of solving problems in mathematics encourages the quality of systematization and an organised way of thinking in daily life situations.

Mathematics has an inherent nature that helps the scholar to imbibe many good qualities like hard work, punctuality, regularity, neatness and cleanliness in his daily life. These habits inspire the students to lead a happy and joyful life. Mathematics is also called as a subject of truthfulness. This means that in mathematics you can't find a variety of answers to a particular problem, instilling precision and truth in the scholar.

Like music, art, and dance, mathematics too gives joy. When the child gets interested in a problem, he does not want to leave it unsolved. He tries, fails, tries again. He learns to persevere. He may lose track of time, forget his hunger.

When, after a long struggle, the solution emerges, the inner joy is immeasurable. Success brings a sense of ecstasy. Archimedes, overjoyed by his discovery of the principle named after him, ran into the street like a child, forgetting that he was naked.

This is the smile that we see in class when a child solves a problem correctly and gets positive reinforcement from the teacher. After such a joyful experience, the interest of the child in mathematics is multiplied.

Through mathematics some physical

aspects come forth. When one engages in solving a problem the restlessness of the body disappears and one forgets the tiredness of the body.

Many fine motor skills are developed by measuring with a tape, balance, protractor, or constructing geometrical figures using instruments.

Beauty, love and joyfulness are psychic qualities. In humans, these find ultimate expression in our arts– Drawing, Painting, Architecture, Music and Dance. To fully appreciate these takes some knowledge of mathematics. Mathematical regularity, symmetry, order and arrangement play a leading part in beautifying and organising the work of arts. Mathematical patterns form the basis of much that we intuitively understand as aesthetics – the interplay of colours and forms, the marvels of Nature, the rhythms of poetry and music, the wonders of the universe…

To conclude mathematics is an integral part of our life. The various relationships between numbers are called mathematics. And these numbers and their beautiful arrangements can be seen in the petals of flowers, leaves, plants, in insects and animals, in the movement of waves, stars, planets, and what not. Mathematics is also a way of communicating with nature and ourselves.

Question to the Mother: For a mathematical problem, sometimes the solution comes quickly, sometimes it takes too long.

Answer by The Mother: Yes, it is exactly that: it depends on the degree of concentration. If you observe yourself, you will notice this quite well: when it does not come, it is because of a kind of haziness in the brain, something cloudy, like a fog somewhere, and then you are there as in a dream. You push forward trying to find it, and it is as though you were pushing into cotton-wool, you do not see clearly there; and so nothing comes. You gather together all the elements of your intelligence and fix them on one point, and then you do not even try actively to find the thing. All that you do is to concentrate in such a way as to see only the problem – but seeing not only its surface, seeing it in its depth, what it conceals. If you are able to gather together all your mental energies, bringing them to a point which is fixed on the enunciation of the problem, and you stay there, fixed, as though you were about to drill a hole in the wall, all of a sudden it will come. And this is the only way.

The Mother

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CHAPTER 03

Initial Mathematics as envisaged by

Jasbir Singh Malik

PART I

A guide for adults on how to let infants and young children explore numbers by themselves

Introduction

Working on the axiom that an infant's reasoning powers begin to develop after he is born, he cannot possibly be ready to deal with numbers and numerals at the age of a few months when such notions are introduced at home.

On entering school at the age of 6 he understands small numbers as '3 apples and 2 apples are like 5 apples' provided they are laid out in 2 separate lots of 2 and 3 apples respectively then they are physically aggregated. This happens because he has eaten, felt and seen, experienced in short, 1, 2...5 apples, but not large numbers of them. And is therefore acquainted with their names and some of their relationships. However, the child does not comprehend the numerals 'two, three and five' and certainly not '3 + 2 = 5' simply because his senses have never experienced and never can experience such statements. To highlight the gap between what the child knows (experience) and is expected to know (reason), a good example is, a Class I child is expected to know 'that zero represents the absence of something'. *[NCERT 'Guidelines and syllabi for primary stage' 2001, New Delhi, page 38]

We know that children do overcome the problem of 'one and one are two' but the majority do not do so efficiently. In any case the question remains, how can they accomplish this with undeveloped reasoning? The answer is they don't. What they do is to bring their intuitive power into play and intuition, unlike reason is developed even before children are born.

None of the thoughts in this essay were generated by *experiments*. They were culled partly through *experiences* with infants and children and partly through discussions with adults. Interestingly both the italicised words are derived from a common source. From our experiences and discussions, the conclusion we arrived at was the hypothesis that **most people, adults and children alike, do not like the subject and are poor at it**. This happens because at the primary level there is a deficiency in the way the subject is taught. We must recognise in the lower classes (a) the student's reasoning abilities are nowhere near the level the concepts in the curriculum require, (b) the younger the student the stronger his intuition, which distances him from reason therefore from the curriculum and (c) the student has his own approach to the acquisition of knowledge which is largely based on intuition. The teacher must acknowledge this hypothesis.

We did not test this hypothesis because certainly since the middle of the last century, perhaps earlier, many others for different reasons also arrived at similar conclusions viz. the quality of mathematical knowledge is poor; the curriculum requires change and the teachers of the early classes are not sure of what and how to teach. Unfortunately the remedies were perhaps ad hoc and certainly did not work, indicating that something was wrong with the testing methods. All this in spite of the fact that some of the investigations were on the national scale in countries which were short of neither funds nor expert knowledge. Perhaps they were hamstrung by history, hierarchy and red tape.

The most well known instance is that of 'New Math'. It was introduced dramatically in US grade schools in the 1960's. There are many thoughts on the reason for the introduction, a popular one being it was in consequence to the launch of the first artificial satellite, Sputnik. This was perceived as evidence that the mathematical skills of Soviet engineers were much superior to those of US engineers; therefore, the US had to catch up quickly. Among the subjects introduced in the first decade of schooling were symbolic logic, Boolean algebra and abstract algebra. This fashion lasted for about 10 years in spite of vigorous protests by parents, teachers and eminent mathematicians/ physicists. Some felt New Math produced students who had 'heard of the commutative law but did not know the multiplication table' [G. F. Simmons in the preface of his book *Precalculus Mathematics in a Nutshell - Wikipedia*].

Others were of the opinion 'first there must be freedom of thought; second we do not want to teach just words and third subjects should not be introduced without explaining the purpose or reason, or without giving any way in which the material could really be used to discover something interesting. I don't think it is worth while teaching such material.' [R.P. Feynman in his 1965 essay 'New textbooks for the New Mathematics' - Wikipedia].

The least well known instance is that of 'Mathematics'. For centuries among the subjects the schoolboy learned was arithmetic (a very old and eminently suitable name for the early study of numbers) followed a few years later by algebra and geometry; either together or a year part. Suddenly the 3 subjects were amalgamated into 'Mathematics' or 'Math' depending on which side of the Atlantic you were on, but there were no changes in content. Why change the name of a product if the customer is unaware of it? It is worth noting until the time of this 'amalgamation' the English textbooks on most aspects of mathematics were first published as much as 100 years earlier and thereafter there had been innumerable reprints with very few revisions.

The changes that have impacted today's school children the most are the changes in the national curricula of many countries brought about by studies indicating that the mathematics results in schools of those countries or the provinces were poor. Of course there is always a flip side to everything; the papers tell us in 2014, Mathematics is the second most popular honours course in Delhi University * [***Relating to India, Britain, Ontario and B.C., I hope to give in quotes with sources, the poor school results which prompted the authorities to commission new curricula which, in the event, were unable to improve the results leading to a fresh curriculum being commissioned.***]

Before we turn the page to the essay itself, you must meet our protagonist the 'infant – child'. In this essay 'infant – child' is a person, male or female, who is less than 7 years old but always has a feminine personal pronoun 'she', 'her', etc. Similarly the person above the age of 7, male or female, always has a masculine pronoun 'he', 'him' etc. This simplifies reading because the personal pronouns indicate whether we are reading about an infant –child or an adult.

There is another important difference between the infant – child and the adult. She has a huge reservoir of some prodigious spirit which enables her to look upon problems and failures as steps to progress, not tasks or events to be dealt with. She solves or resolves them. Success is not connected to smugness or arrogance or honour, nor is failure connected to humiliation or disgrace or fear. The job is to be done and 'done' means, 'to the best of one's ability'.

We, adults, do not have this spirit may be because she allowed it to fade away by the time she became one of us. Perhaps this may not happen if instead of waiting for her to become an adult, the adult makes an effort to understand the infant - child.

Experience is necessary for early comprehension. Early experience is supported by intuition. Early mathematics curriculum needs alteration. Conclusions not tested because there is a universal problem. New Math discarded 3 books replaced by 3 sections of 1 book. Changes in curriculums were not successful. Infant – child. Job must be well done, happily. Understand the child.

The Initial Mathematics (IM) Approach

The Cover Story of The New York Times Magazine of July 27 2014 is titled, 'Q: Why does Everyone Hate the New Math? A: Because no one understands it – not even the teachers. The subtitle is, '(New Math) – (New Teaching) = Failure' and a blurb reveals, ''The Common Core is the best way to teach math but no one has shown the teachers how to teach it."

The current approach to teaching mathematics to the infant-child is inconsistent with her psyche and available faculties that can result in a poor understanding of the subject. To address this, the teacher must:

1. change his behaviour {towards the infant-child} by including in it the infant-child's approach.
2. not use his prior knowledge of numbers as a basis for teaching.

1. Behaviour Changes

Infant – Child's Approach	Parent's / Teacher's Approach	IM Approach
To acquire knowledge, the infant-child uses 3 resources or faculties that are available to her: 1. intuition, which is innate and is fully developed in the infant-child. 2. experience, which is developing as it is acquired. Her first breath is her first experience. 3. reason, which is being developed.	The adult only uses reason as the means to teach Maths to the infant-child. Using the least developed faculty (from the perspective of the infant-child), results in an inefficient approach to acquiring knowledge of mathematics.	The teacher must: • honestly acknowledge that his approach to teaching is different, since it is reason biased/ based. • since the infant-child only has intuition as a resource to acquire knowledge, the infant-child must be given an opportunity to experience real-life situations through her senses. This experiential information will add to her intuitive abilities to enhance her knowledge of IM. {There should be a mechanism to assess these abilities of the teacher.}

The infant-child has complete trust in the teacher because she comes from a place where she is aware of faith, confidence, dependence, veracity etc.	We are raised to ensure we do not hurt the feelings of others. Teachers sometimes when confronting their students regarding a problem, say untruths to avoid hurting the child. But the child who is trusting and is highly intuitive, senses that all the relevant information has not been made available, resulting in a potential loss of understanding. Over time, repeated situations like this create multiple gaps in the infant-child's understanding of the subject along with a decline in the trust in the teacher.	If the teacher is trying to address a problem the infant-child has, then the teacher must truthfully explain the cause of the problem even if it means causing some distress to the infant-child. In the event, it may not cause distress because what happens meets her expectation. This approach will also reinforce the infant-child's trust in the teacher.
The infant-child comes from an environment that is friendly and welcoming.	The adult tends to assume the infant-child knows nothing and therefore speaks 'down' to her.	The teacher must: • recognize that he has something to learn from the infant-child before he can teach. • learn to take his cue from the child – the infant-child is the 'king' or customer.
The infant-child enjoys making mistakes because it provides a stimulating variety to the learning process.	The adult dislikes them and tries to aggressively minimize them.	The teacher must recognize that making mistakes are an essential part of the learning process. The 'mistake' gives the teacher an excellent opportunity to know what is bothering the child.

The infant-child constantly makes judgments. For example she'd rather grab 2 sweets instead of one.	The adult believes it is correct to be non-judgmental. Yet he divides their work as either 'good or 'bad'; 'right' or 'wrong'.	The infant-child's approach is natural i.e. nature distinguishes 'right' from 'wrong'. The adult on the other hand uses a 'civilized' approach and tries to be non-judgmental with the infant-child. The adult must recognize the infant-child's approach, not suppress it but at the same time teach the civilized approach according to the infant-child's age (i.e. this cannot be taught to a 3-year old).
The infant child often must struggle to solve a problem.	Instead of allowing the child to struggle when solving a problem through exploration, the adult often sees the infant-child's distress and prematurely solves the problem for the child.	The adult should not interfere with the infant-child's struggle because the child is exploring.
The infant-child regularly uses facilities in everyday life whose inner workings she does not understand.	The adult regularly uses facilities in everyday life whose inner workings he does not understand e.g. an automobile. Yet he insists that she understands everything he teaches.	The teacher should allow the child to explore and take the time needed for eventual understanding.
The infant-child is born without fear because she comes from a uniformly stable environment, the womb.	The conventional approach to teaching makes the infant-child fearful because {her present environment is neither uniform nor stable with uncertain results generating anxiety}her answers and actions 'are (i) classified and (ii) generate emotions.Neither happened earlier. This causes anxiety, then fear.	{Fear can be removed through genuine laughter i.e. infant-child and teacher must share equally in the 'joke'.} Fear can never be removed. It has been institutionalized. It has been a weapon for centuries. The best the teacher can do is to be aware that he does generate fear and TRY to avoid it.

2. Knowledge of Numbers
Through prior knowledge and experience, the adult knows the outcome of the mathematical problem. He uses this knowledge (the outcome) as a basis for teaching. His teaching approach is therefore influenced by the known outcome.

Infant – Child's Psyche	Parent's / Teacher's Approach	IM Approach
The infant-child sees each of the initial numbers as independent wholes that are not connected to each other.	The adult sees them as part of an ordered series.	To enable communication the teacher must give each number a name and let the child discover the order for herself.
The infant-child sees number as an adjective and not as a noun because number does not exist in nature as a noun. Number is a qualifier i.e. an adjective. But in order to work with numbers, the infant-child must regard them a noun. So in her wisdom, she creates them: 'one car' becomes 'onecar', 'two cars' become 'twocars'. And 'twocars' is not the result of a summation of 'onecar' and 'onecar'. Also, the 'onecar' and the other 'onecar' are unique, they have nothing in common. However, if she has seen one car in the large garage and watches another car drive in, she will expect to see two cars in the garage. This 'addition' is intuitive, limited to small numbers, involving no knowledge of counting or of the relationship between numbers.	The adult sees numbers clearly as a noun which results in a conflict in the infant-child. The language dictionary as opposed to the mathematics dictionary, suggests that number is a symbol or word or a group of either of these showing how many or what place in a sequence. Recalling that the infant-child's window is intuition, 'symbol', 'word' and 'group' which are creations of reason not of nature, are unintelligible to her and will remain so until she has a large enough experiential inventory to enable her to move from the concrete to the conceptual.	Regarding 'onecar' and 'twocars', it takes the infant-child hundreds of experiences before she begins to extract the 'one' from an 'onecar', 'oneman' or 'onetree'; or the two from 'twobottles', 'twocars' or 'tworattles'. This is the first step towards the number concept. It takes yet another lot of experiences for the relationship between one and two to emerge. Only then has the little girl taken the first step towards counting. She counts or puts numbers in order after she is introduced to them, just as the adult puts things in order only after he knows all of them.

Just as addition and subtraction can be actually seen, so can multiplication if it is introduced as repeated addition.	The adult uses multiplication tables as means to teach multiplication. But the infant-child is confused as she lacks any context or understanding of its purpose.	To enable the child to experience multiplication she must be allowed to experience repeated addition through real world illustrations.
Precision with numbers takes time. For example, the infant-child distinguishes immediately the difference between 1, 2 and 3. But does not discern the difference between 3 and 4, 4 and 5 etc. until she has a large bank of experiences of such entities.	To the adult precision with numbers is essential. Though he accepts imprecision in language. For example, lavender, mauve and purple are all purple to an infant-child and the adult is willing to accept this response from her.	The adult must accept imprecision with numbers especially for the younger child. This approach will help him communicate with the infant-child more effectively.

PART II

Introduction

This essay seeks to explain why mathematics is not liked by most of us, children and adults alike and suggests a remedy.

Working on the postulate that the child has no reasoning power at birth and it takes a couple of decades to peak, his ability at the age of 3 or 4 years cannot possibly be adequate to deal with numbers and numerals when they are introduced at home and school.

We know that little children do overcome this problem but the majority of them not efficiently. How does he do it. Perhaps the infant has some innate ability other than reasoning which makes him give you a wider smile when you give him a second balloon. This essay is about that part of mathematics which enables the infant and young child to do mathematics without reasoning ability.

The thoughts in this essay were generated partly by interactions with infants and children and partly out of discussions. It is generally experiential. There is very little here generated by the rigours of logic and science. Whatever was experienced was not put to any statistical testing. We hope there is agreement that most little children are therefore the adults they become. Later in this essay there is another conclusion arrived at without any testing but the redeeming feature here is the sample is as large as the population.

For ease of reading and reference, 'infant – child' will stand for a person who is less than 6 years old and is a girl to distinguish between 'he', an adult and 'she', our protagonist. We hope this simple differentiation will not be looked upon as the drawing of battle lines between ourselves and the 'little brats'. Can one enter a time machine, go back 15, 25, 35, 45…. years and battle with a 5 –year old oneself?

Intuition

Most adults rely on reason; all infant – children from birth or even earlier rely on intuition.

Intuition cannot be defined because it does not depend on logic or words. It enables us to arrive at an instantaneous reasoned conclusion without the use of reason.

This is significant because reason needs words to formulate the question, to process it and then to tell us "why" which limits us to our understanding of those words. On the other hand, intuition does not tell us "why" instead it shows us what to do. Meaning becomes a realization not a reasoned conclusion.

A batsman has been well instructed on what to look for, the pitch, the field placement, the wind, the humidity and so much else, before deciding on the best shot. However, when he actually executes the shot he does not reason out his stroke based on the words he has heard. He plays without conscious thought. He uses intuition. Intuition helps him take advantage not only of the instructions but also the experiences he has accumulated. If he had no instructions and no experience of cricket, his intuition would draw upon cricket-like experiences to enable him to put up as good a performance as possible.

Infants depend fully upon intuition, therefore one expects to find at least a little of it in adults but this does not happen because as the infant grows older she is told with increasing emphasis that the brain, therefore reason, is the only reliable window for gathering and using knowledge. Reason helps her understand everything. This happens in spite of the fact that we all know the likes of Einstein, the eminent scientist, on the one hand and Tendulkar, a world-class cricketer on the other, come rarely. They have received no special instruction.

For intuition to arrive at a reasoned conclusion without the use of reason, the mind cannot be a blank slate but must have an alternate mechanism to gather, comprehend and use knowledge.

Intuition does not operate on a blank mind.

The following story and a little introspection may help understand the impact of intuition on the problem-solving process:

In 2011, IBM built Watson, a computer, to beat the two best players of the popular quiz programme, Jeopardy. At a show, the display of its extraordinary abilities caught the attention of a New York Times reporter who raved about the achievements of artificial intelligence. Dr. Michael Brown, Professor, Department of Mathematics Statistics and Computer Science at Simmons College, Boston read the NYT article and wrote to the Editor that the computer "…begins by using logic. But when mathematicians, scientists and other experts at problem-solving are studied and asked to elucidate their problem-solving processes, they regularly report that they work first in intuitive images, irreducible to logic and often visionary or dreamlike. It is only after they have 'seen around the corner' that logical reasoning is engaged, to systematize the findings and make them coherent."

To paraphrase, the framework of the solution is initially not derived through reason but an ability called intuition. Reason is applied later "to systematize the findings and make them coherent." So, even adults who are confronted with difficult problems to solve, engage intuition to help them look "around the corner" to see the solution.

Prof. Brown ends his letter with, "The ultimate challenge for computer science is not artificial intelligence but artificial imagination".

So if intuition, an integral part of the natural state is available from infancy, why is it that adults disregard it and rely instead on the tools of reason to teach problem-solving to young children?

Mathematics is two Subjects

At birth the infant accesses the world solely through intuition but with time its extent diminishes and is replaced by reason. By the time she is 7 or 8-years old in Class 2 or 3, she relies largely on reason.

Considering mathematics as a product and the teacher as a salesman, we can divide his customers into two independent sets, the infant-child whose window to numbers is intuition and the older child including the adult in University whose window to numbers is reason. The older child inherits her knowledge of numbers from the infant-child.

As the teacher does not acknowledge the existence of intuition, the infant-child is taught her numbers through reason. Obviously, this is not fair. With two different classes of customers using completely different windows to acquire knowledge, we need to introduce a new discipline, Initial Mathematics (IM), for the infant-child, which would have a fresh pedagogy based on intuition. Of course, parents and teachers must accept that intuition exists, that it is not a bad thing, that the infant-child has thrived on it and all forms of creativity depend on it.

Mathematics is two subjects: the one with pedagogy based on reason, the other on intuition.

Besides having separate windows to acquire knowledge of mathematics, the two customers have separate approaches. The hands of both the infant-child and the older one are held and they are taught to "learn" mathematics. This is the traditional approach. However, if left to herself the infant-child would spontaneously and joyfully not 'learn' but 'uncover' the entire panorama of IM and so, over time unfold one element of the whole after another.

When the traditional approach is applied, the infant-child's dislike arises because the adult teaches number as concepts which she does not understand; and to complicate things further she has a niggling feeling that perhaps she already knows something similar. The adult writes '1' on the board and says 'This is one.' Then he writes '2' and says 'This is two.' Now consider the adult's actions and statements from the point of view of the infant-child.

She is unhappy and thinks, "How on earth can the sound 'one' and the symbol '1' be the same?" Her unhappiness grows to become confusion.

The confusion is compounded when the adult counts: 'One, two, three….' And insists on the sequence, first "one", then "two" and then "three" which to him is the beauty of order. To the infant-child, with no idea of magnitude or value, "one", "two" and "three" is no different from "two", "three" and "one".

Concepts and their subordinates, i.e. symbols, play a pivotal role in mathematics but as well-wishers of the child we must not forget that both concepts and symbols are creations, It is only the concrete that the infant-child knows. Her mathematics is not Ramanujan's mathematics of the abstract instead it is the mathematics of the concrete, Initial Mathematics (IM).

Numbers and Numerals are Concepts and Icons Unknown to the Infant-child

The word 'number' means many things to the mathematician but none of them stem from intuition, therefore we will not examine any of them. Instead let us spend a few moments on what number means to the infant-child.

The language dictionary as opposed to the mathematics dictionary, suggests that number is a symbol or word or a group of either of these showing how many or what place in a sequence. Recalling that the infant-child's window is intuition, 'symbol', 'word' and 'group' which are creations of reason not of nature, are unintelligible to her and will remain so until she has a large enough experiential inventory to enable her to move from the concrete to the conceptual.

So we come to the conclusion that the infant is unaware of numbers but we know she is not an illiterate so number must feature somewhere in her understanding and indeed it does.

Behold, the 'compositeword.' The infant-child does not see one car or two cars, instead she sees 'onecar' and 'twocars'. And 'twocars' is not the result of a summation of 'onecar' and 'onecar'. Also, the 'onecar' and the other 'onecar' are unique, they have nothing in common. However, if she has seen one car in the large garage and watches another car drive in, she will expect to see two cars in the garage. This 'addition' is intuitive, limited to small numbers, involving no knowledge of counting or of the relationship between numbers.

Another example may clarify this better. Next time, you say goodnight to a 1-year old, put 2 éclairs under her pillow and tell her she can eat them after she brushes her teeth the following morning. When she is asleep remove one éclair. Is the loud protest the following morning the result of a mathematical subtraction? No, it is just the identification of a totally unexpected event.

Coming back to 'onecar' and 'twocars', it takes the infant-child hundreds of experiences before she begins to extract the 'one' from an 'onecar', 'oneman' or 'onetree'; or the two from 'twobottles', 'twocars' or 'tworattles'. This is the first step towards the number concept. It takes yet another lot of experiences for the relationship between one and two to emerge. Only then has the little girl taken the first step towards counting. She counts or puts numbers in order after she is introduced to them, just as the adult puts things in order only after he knows all of them.

The little girl's secret weapon of the compositeword is not limited to numbers. It applies to all qualified nouns. 'redhat', 'redribbon', and 'redcrayon' result in the extraction of redness. 'hungryboy', 'hungrydog' and 'hungrylion' give hunger.

A tangential thought: Question addressed to a 2-year old, 'How do you know you are thirsty, if you do not know what thirst is?'

Teaching and learning may not be two sides of the same coin

The two aspects to tutoring, 'teach' and 'learn', are generally looked upon as the two sides of the same coin. The student learns when the teacher teaches that Oslo is the capital of Norway or that tomorrow will arrive immediately after 12.00 tonight but how does the teacher teach 4 + 2 = 6? You can help her learn but can you teach it? Some aspects of knowledge can be taught but others can only be acquired through one's own exploration.

Some coins have only one side

The adult must gracefully acknowledge that there are times when his presence is not required, that his help can retard the learning process. Not only must the infant-child be allowed to make mistakes but it must happen without fear and she must be given time first to acknowledge her mistakes, then to struggle to rectify them and finally to savour a job well done. All this while the adult learns to watch and play the fiddle.

A tangential thought: Should teachers report on students or vice versa?

Another tangential thought: It is very, very difficult for a parent to twiddle his or her thumbs while watching the child make mistakes and more so to watch the child struggle to rectify them.

The infant-child is not taught yet learns language

On moving out of the womb, the infant is assaulted by a large variety of sensory experiences and all of them are new. Lesser beings would happily allow themselves to be overwhelmed but not our protagonist. She tells herself, first she must get to know where she is and to do so she must do 3 things simultaneously: begin observing this totally unfamiliar environment, recording her experiences and recognizing the context in which they take place. Only then will it be possible to begin establishing relationships between her experiences and her environment.

Having reached thus far, she realises that she does not know the names of what she is observing nor can she record anything because she does not know how to write. Even if she knew her a, b, c, she would not be able to proceed because she can verbalise neither her thoughts nor her feelings. To cap it all she has no framework to make comparisons. Both her dictionary and her diary today, are fat books of blank pages. She does not know this is sweet and that is sour; this is a man and that was a dinosaur; this smells and that reeks; should we break rules or should we break glass; what is to communicate and what is meaning and…

Then the lights fade and a few seconds later they come on to reveal a changed scene. She is laughing from the pit of her stomach because her intuition has been working overtime and she has started to treat these peculiarities, as intuition requires them to be treated. Funny oddities to solve! Not difficulties to fret over! Oddities do not intimidate her. They stimulate her and she enjoys their challenge. The little girl has a wonderful approach to life and its problems. She does not allow herself to be overwhelmed; instead she rolls up her sleeves and makes the finding of solutions an enjoyable game. There is no room for fear in the infant's life. After all success and failure are concepts, therefore unknown to her.

To compare the infant's plight when she suddenly finds herself in an utterly foreign world outside her mother, let us consider a prosperous middle aged man who last night celebrated the payment of the last installment of the loan he had taken for his 4-bedroom apartment with a quiet 5-star dinner with his wife and is now contentedly brushing his teeth when suddenly the bathroom crumbles, the brush and the toothpaste vanish and he stands naked in the open under a green sky, in completely unfamiliar surroundings of violet and crimson rocks streaked in yellow and black. There seems to be a lot of activity but nothing moves. The smells are acrid and overpowering. Every thing he touches is jagged and rough. There is a variety of sounds but they are utterly unfamiliar and not one is pleasant. His mouth is half open, his head is inclined back, his arms are raised and the open palms are held out to ward off missiles which do not exist. His eyes are wide open. He is lost, his mind is a blank and his body is immobile. He is terrorized.

We find the infant confident, in control and tackling problems in an adult manner whereas the adult, in similar circumstances, is a bundle of nerves, completely lost and tackling problems in a childish manner.

Thanks to her intuitive approach, in a few weeks she has a working knowledge of speech and is able to communicate in a rudimentary fashion and in a little more than a year she is able to converse.

Let us not forget all this has happened in spite of the complete absence of instruction and a starting point of no experience and without any knowledge of language. The crowning achievement arrives a few months later when she has embraced the simpler rules of the adult's language and has started to incorporate his grammar and syntax in her speech. All this by an infant-child, who started life virtually as a mute. Then unlike the mute, without help, this ordinary infant-child, without any special endowment, is able to speak.

This is a remarkable success story and must be examined:

- the infant listens to the sounds she hears
- assesses them
- sorts them with the help of her intuition into three compartments (a) language, (b) the chirping of birds, the slamming of doors and similar sounds and (c) others for which her intuition has not as yet got a group heading
- retains (a) and (b) in her Class I memory and (c) in her Class II memory
- recalls and assesses (a) and repeatedly reassesses it against her developing knowledge of words and language and continues to do so on an on-going basis
- creates her own language
- tests it
- accepts the need to conform because she has no desire to assert her individuality or her independence
- acknowledges her mistakes without losing heart
- uses the language she has created
- the sounds in compartment (b) of birds, doors and such like things are treated similarly over time and those in compartment (c) are kept intact for investigation later yet

For an adult to do all this, would require a variety of postgraduate degrees, a need to create data, the preparation of written records, the creation of filing space, employment of an assistant or two and finally to overshoot at least two planned dates of completion.

The little girl manages this Herculean task because there is virtually no intervention therefore no confusion between intuition and reason. Her motivation is from within, pure enjoyment. She enjoys exploring as much as she enjoys being lost. She is not scared of making mistakes because they quickly show her the straightforward correct path. Lastly and importantly she has no problem with understanding because her window is intuition, which takes her instantaneously from not knowing to knowing. She does not have to work her way through the path of reason. In her language learning mode she has her wits about her, displays indomitable spirit and is driven by a passion to know.

Let us see how the adult “teaches” language to the infant-child.

He is besotted by her. Lets her take charge and is delighted to be a spectator. He does not correct her; instead he repeats her mistakes encouragingly, to the extent of incorporating her errors in his speech. All her approximations are acceptable – choice of word, grammar, pronunciation, everything. In the absence of teaching there is no syllabus and no set time limits. At times it may appear that the infant is floundering or has lost interest but such occasions are brief. She returns vigorously and resumes control. The initial learning of language takes place with minimal, if any, teaching by the adult.

The daughter is the mother of Man.

The infant-child is taught but finds it difficult to learn numbers

In the last paragraph of the previous section we read the adult made no attempt to teach how either words or groups of words are strung together to create language. He did not teach because he was besotted by the little girl; but even if he were not, he would have been unable to teach because communication between them was impossible in the absence of a common language. This suited the infant - child admirably because it left her alone to explore and learn the teacher's language entirely by herself, by her rules and at her pace.

These observations are worth a little reflection. Suppose the new born baby did not take the initiative and at the end of three months the adult ceased to be besotted. If this happened the adult would very soon start teaching, but in which language? There is no language which both of them know. As a matter of fact, the little girl does not know any language at all. To change matters, the baby assumes leadership, works on her own at learning the teacher's language and when she achieves a certain level of excellence she hands herself over to him. Graciously, she anticipates his need.

Strange as it may seem, if language is to be taught by adults, it can happen only after the infant - child has learned it by herself up to a certain base level!

Number has a lot in common with language and for efficient learning, a number language also has to be built up in advance by the infant-child. One way of doing so is for her to eavesdrop on a myriad 'number' conversations. What could be better than listening and watching, innumerable customers placing orders, a parent or a sibling executing the orders involving weighments and then carrying out a monetary transaction for each order?

True, this number listening and watching is much more involved than what the newborn baby did with language. But let us not forget, the infant-child is so much older when learning numbers than she was when learning language.

Younger or older, very few infant - children have parents or siblings selling peanuts on the pavements of a metropolis, resulting in most infant-children having to confront a teacher teaching numbers without the benefit of a number language under her belt. And that is a tragedy of mammoth proportions.

This is what happens when an infant - child has to face a teacher teaching numbers without the cushioning - benefit of a number language:

He believes her mind is a clean slate, when in fact intuition has been scribbling on the slate from the time she was in her mother's womb. The teacher's approach is foreign to her, which means tension, but she does not know what tension is. All she knows is that she is uncomfortable, therefore confusion. On the other hand, the teacher does not see her problem and is blissfully happy with his clean slate. Therefore, a lopsided relationship.

- He assumes her vocabulary is larger than it is in fact. He also assumes she has some reasoning powers e.g. when he teaches counting he assumes that she sees order in one, two, three… also realises the difference between two consecutive terms of the series is always the same as the first term of the series. She would discern order if she were shown a large number of different series and if she knew what 'order', 'magnitude', 'first term' etc. meant. The result is a slightly more lopsided relationship.

- He encourages her to strive to avoid mistakes. If she understood that thought, she would explain, 'I enjoy doing my work and sometimes I realise I have made a mistake. When that happens, I redo the relevant part and I find the mistake is no longer there.' In essence, 'a mistake' means different things to the teacher and to the student. To the teacher a mistake is not a nice thing but to the student it is part of the game. The relationship gets to be yet more lopsided.

- He believes discipline must be enforced in class. If she understood that thought she would respond with, 'Surely for discipline to be discipline it must be self - enforced'. Her proof would be the devotion with which she attacked the language issue, all by herself. To the teacher, discipline must be enforced; to the student discipline is self-enforced. Yet another load of confusion for the little girl to deal with.

Other instances, which add to the confusion of the lonely girl, are:

- Though not in pristine condition the school-going infant-child is still heavily inclined towards learning by exploration rather than by instruction.

- She understands the concrete and is only just beginning to discern the very occasional glimpse of the concept.

- The adult believes unpleasant things must be avoided and we must always work towards minimising their impact. This is not what the infant-child believes. When she stepped out of her mother's womb her lexicon did have words such as mistake, difficult, unfair, hate, anger etc. but they do not do the same things to her as they do to the adult. To her they are things, which must be faced and overcome; she sees no point in trying to avoid them or looking upon them with distaste. Problems arise when the adult gets angry and the little girl picks up his anger in spite of his efforts to play it down. Problems arise because he does not know that she knows.

Trust is something we are all born with in good measure but age erodes it at an accelerated pace. Since the adult has the advantage of hindsight one should expect him to acknowledge the little girl's 'handicap', and not let her suffer for it. For old times sake, squeeze out some trust for a lonely child.

Is it any wonder then that numbers generally leave a child with very unpleasant thoughts? This tends to support the findings of an AP - AOL News poll conducted in 2005 in the United States.

About a quarter of the population studied, said mathematics was their favorite school subject, which was about the same number that preferred English and History. However, almost 40% of the 1,000 adults surveyed said they hated mathematics in school. The poll also found that twice as many people said they hated mathematics as compared with any other subject. A fourth finding of this poll showed that the proportion of women who disliked mathematics was higher than that of men.

http://www2.ljworld.com/news/2005/aug/17/poll_shows_americas_lovehate_relationship_math/

The daughter is not the mother of Man

How the infant-child would like to learn numbers

We have seen the infant-child pick up her knowledge of language without any assistance whatsoever. She discovered language merely by observing and listening.

We have also seen poor results when something is taught: numbers.

Let us see what happens when numbers are not taught. Instead can the infant-child acquire her knowledge of numbers in the same way that she acquired her knowledge of language i.e. merely by observing and listening?

During the cold months, on the streets of Delhi, 8-9 year old children can be seen selling peanuts at makeshift roadside stalls. Here is one which was set up in the morning next to a bus stop by an 8 year old's mother and after managing it for half an hour she left to set up another stall 500m up the road to be managed by her 12 year old, assisted by the youngest sibling.

A 1.5m x 1.5m piece of gunny spread on the pavement marks the stall. Two smaller pieces of gunny, a pile of envelopes in 3 sizes made from old newspapers and a pair of weighing scales with 25g, 50g, 100g and 200g weights (one of them being an aggregate of a stone, a rusty piece of iron and a couple of coins) completes the stall. The pyramid of unshelled nuts lies on one of the pieces of gunny and the child sits behind the pyramid on the other piece with the earnings and the morning's capital stashed under this 'mat'.

The young person takes the order, weighs the nuts, puts them in the appropriate paper bag, mentally calculates the amount, announces it to the customer, hands over the nuts, collects the cash, does a mental subtraction and hands back the change.

Assuming she works for 4 or 5 hours in the day she must be responsible for more than 100 daily transactions. We must remember the young person has neither been to school nor received instructions on how to manage a stall and a comment on her day's labour is limited to those occasions when the day's earnings are less than the parent's estimate. The comment is perhaps no more than a tight slap.

Without the help of teaching, the infant-child has learnt the basics of numbers simply by listening and observing adults as they carried out their day-to-day chores.

And this is exactly how she picked up her knowledge of language.

Based on the infant-child's ability to acquire knowledge of language and observations such as the illustration above that have been informally tested in the classroom through personal experience, the following hypothesis emerges:

- The infant-child has an innate ability to acquire knowledge of numbers through observation and listening.
- Attempts to teach numbers to the infant-child without giving her enough opportunity to explore are unsuccessful.

Conclusion

Innumerable times in the course of a day the infant-child has a language within earshot so she is repeatedly exposed to a large variety of words, pronunciations, constructions and all the other aspects of language.

Just exposure to a variety of words and a variety of interactions between words, magically results in the infant-child acquiring proficiency in language without help.

This is also true of numbers. The infant-child needs to be exposed to a variety of numbers and a variety of relationships between numbers. Without any other help from adults, this exposure by itself will lead to proficiency in initial mathematics and eventually to mathematics. Unfortunately, in day-to-day life such exposures to numbers and their relationship happen very rarely, an example being the peanut seller on the streets of Delhi.

Initial Mathematics (IM) is a form of "teaching" with its own pedagogy and is based on the hypothesis discussed in the previous section. The "teacher" must realize that for IM to be successful, he must facilitate rather than instruct. The "teacher" must take his cue from the child. This will allow the infant-child to naturally explore, experiment, make mistakes and enjoy the experience.

IM allows the infant-child to grow into the world of concepts that make up traditional mathematics. This is because IM acknowledges the infant-child's innate awareness of 'number' and transforms this awareness to an understanding of the concepts that are part of traditional mathematics.

The infant-child's understanding of 'number' develops from her innate awareness of 'Compositeword'.

After graduating from IM the infant-child grows to become a child, which will typically happen after three years of IM at the age of seven.

For IM to be a success, situations like that of the peanut seller have to be created and made available to the infant-child and in the next section we see how this is accomplished.

Contents

A. Mistakes: Generally, when adults make mistakes, they tend to look upon them as something undesirable. Instead, adults, including teachers should take their cue from the infant-child who we have seen looks at mistakes as learning devices. As soon as she realises her error, which may happen well after she thinks she has completed whatever she set out to do, she checks her work and rechecks it until she discovers what went wrong. The discovery brings with it a sense of joy, which substantially reduces the probability of repeating the mistake or indeed making other mistakes. In the event of her missing out on identifying her error, the teacher should intervene but limit his intervention to suggestions such as, she should check her work.

B. Trust: The infant-child has had little experience of life so tends to offer her complete faith to all those with whom she has relationships. On the other hand the adult because of his experiences does not begin a relationship with complete openness. A teacher must remember that he is the infant-child's Guru and mistrust can result in the creation of doubts in her mind and affect the Guru's credibility. He must return her trust unconditionally! Trust for the child includes the belief that the child will perform to the best of her ability yet may not achieve the objectives established by the school or the curriculum.

C. Repetition: Repetition is not a bad thing. The infant - child studying subjects involving skills or the arts, spends a good part of her study time practicing and this is found to be quite acceptable. However, if the word 'practicing' were to be changed to 'repeating', the parents would wonder at the ability of the teacher. We practice or repeat to gain experience. Experience and Intuition are birds of the same feather. Intuition uses Experience because it enlarges Intuition.

D. Laughter: Laughing and smiling are parts of the infant–child's repertoire of expressions and she feels comfortable when others express themselves similarly. In any case laughter is a useful leveler especially when the one with whom the infant–child is interacting is thrice as tall and 5 or 6 times as heavy as she is. One belly laugh a day is a good prescription to balance the relationship. Yoga experts think so too.

E. **Anger:** The little girl does not think anger is something that should not be displayed. When she feels angry she lets you know, immediately. Actually, she gets angry because she wants to tell you something. Surely, there is no point in getting angry if you are going to keep it to yourself. Also, she gets confused and mystified if she sees somebody smiling when she expects him to be angry.

F. **Judgement:** Should we endeavour to keep the balance of Justice always balanced and level? Declare that one of the parties involved, is indeed wrong. As a fellow being, it is our duty to correct, otherwise our benevolence results in wasting resources and we may force a non-intuitive assessment of the unknown upon the infant and introduce her to fear.

G. **Jargon:** Like all of us, the infant – child is at ease with familiar things and uncomfortable with unfamiliar ones. Little can be done about an unfamiliar experience but something can be done about an unfamiliar word. 'Addition' is a word she has not encountered so it should be substituted by a known word, 'together' or 'all' or even a gesture of slightly cupped palms slowly brought together. 'Put together how many stones would you have?' or 'the naughty boy wants all the leaves.' There is nothing unfamiliar about these words, there is no jargon. Later, 'addition' can become an unobtrusive synonym.

H. **Exploration and Experience**

I. **Encouragement/Praise**

J. **Child must struggle**

K. **Social**

L. **Question**

M. **Estimation**

N. **Arrogance**

O. **Intuitive Awareness and Reason**

P. **Curriculum: all children must pass the tests**

Q. **Understanding and Revealing**

R. **Learning Measured in Time**

S. **Whole Vs Part**

T. **Truth**

Activities

Activity 1

Objectives

1. To experience numbers by seeing relationships (subtraction) between them.
2. To create a vocabulary of 5 names of numbers to enable communication.

Materials

20 each of 6 different small objects say stones, counters, marbles etc.

Method

a) Lay down 5 stones in a cluster.

b) Pointing to the cluster, say, 'I picked up 5 stones from the park and put them here.' Do not just say, '5 stones.' The infant-child acquires knowledge of her surroundings because she experiences them and no experience can be described by, '5 stones.' An experience takes place when one does something or something is done to one.

c) Pick up 1 of the 5 stones and place it clearly apart from the others and pointing to it, say, "1 stone decided to roll away because he wanted to be alone." Then pointing to the remnant cluster of 4 stones, say, "The 4 stones which were left alone waited for that stone to return."

d) In the course of the day repeat a. to c. 3 or 4 times with other objects. To ensure further variety, the 'story' must be different and the single object placed at different locations.

e) Over the next 5 or 6 days repeat the activity about 15 times, each time with a fresh 'story' and removing a different number of objects. Note, you will introduce 'None.'

f) Do not follow any order. It must be left for her to discover.

g) Repeat a. to d. once a week for 2 or 3 months with progressively larger numbers. Fewer repetitions will be required as the infant child progresses.

Test

Individually, taking the infant–children, the teacher will know he has succeeded when he gets the correct answer, orally, with a smile from the infant-child to the oral question which begins with the words, "The hen placed 5 eggs……..." If the answer is wrong, repeat the activity after a couple of days. If all children do not give the correct answer please consult another practitioner.

Activity 2

Objectives

To experience the enlargement of the number vocabulary from 5 to10.

The first activity was up to 5 because infant–children are comfortable with numbers at that level. This activity is to give them the opportunity to experience larger numbers. As a first step, we restrict ourselves to just five more.

Materials

20 each of 6 different small objects say stones, counters, marbles etc.

Method

Lay down 7 stones in one row. Over the next few days place them in 2 rows then 3 rows. Next, form different numbers of clusters. 1 cluster is the most difficult. It will not be long before the infant–child senses more or less immediately clusters of about 10 objects. For larger groups give time.

Repeat as in Activity 1.

Test:

As in Activity 1

Activity 3

Objective

To experience the enlargement of vocabulary from 10 to 20.

Materials

20 each of 6 different small objects say stones, counters, marbles

Method

As in Activity 1 and 2.

Test

As in Activity 1 and 2

Activity 4

Objective

To experience the addition of numbers.

Materials

20 each of 6 different small objects, say stones, counters, marbles etc.

Method

a) Lay down 7 erasers in one group.

b) Pointing to the group, say, "I bought 7 erasers and put them here."

c) Lay down another group of 5 erasers about 20cm away from the first group.

d) Pointing to the second group, say, "I bought 5 more erasers and put them here."

e) With your hands, draw the 2 groups together, cover them with your hands and ask, "How many erasers are there under my hands?"

f) Whether or not you get the correct answer, repeat a. to e. twice with different objects.

g) Repeat a. to f. with different objects with gaps in time as in Activity 1.

Test

As in Activity 1.

Note: An activity on addition of numbers in the decimal systems will be covered subsequently after further exploration and experience of numbers.

Activity 5

Objective

To experience the counting of numbers.

Materials

Varying numbers of many different objects, say differently coloured marbles, gems, trees, 2cm X 3cm pieces of paper etc.

Method

a) Ignore the children, sit at your desk and take out from your pocket a handful of marbles. Muttering to yourself and wondering where the red marble has gone, find it and place it to one side, saying reasonably loudly, "One marble". Again muttering, look for the green one, find it and place it next to the red one, announcing, "Two marbles". Carry on until you have 4 or 5 marbles of your choice, tie them up in a handkerchief and put them in your pocket. Then tell the class that after school you will be playing in a Marbles Championship.

b) Drop another marble into the bottle, saying, "Now there are 2 marbles in the bottle." Again look at it for a moment.

c) Carry on until all 20 are in the bottle.

d) Repeat a. to c. twice with different objects.

e) Repeat a. to d. after a few days to complete objective. Hereafter for objective 2 of activity 1 only.

f) Ask the infant–child if she knows which of the 20 marbles in the bottle was the first to be dropped. She will answer in the negative.

g) Ask her to take the marbles out of the bottle and lay them in a row.

h) Ask her to point out the first marble she laid out? The second? The third?…The twentieth?

i) Repeat a. to h. after a few days to complete objective 2 of activity 1.

Tests

Neither of the objectives of this activity have tests. If the infant–child remains unsure, further experience is all that can be recommended.

Activity 6: Ordinal numbers

Objective

To experience the multiplication of single digit numbers (tables).

Material

20 each of 2 differently shaped leaves and 4 other small objects, say stones, counters, marbles and eras

Method

a) This activity should be broken into 2 parts because the infant–child may not be able to give of her best at the first set of sittings from 2 X 1 = 2 to 2 X 10 = 20. For the first few days, we should restrict ourselves to 2 X 1 = 2 to 2 X 5 or 6. Once this is done, take the next step after a gap of a week or so and complete the table.

b) Before the class begins, prepare the pattern shown in the 4 or 5 lines formed by the shapes () in e. below and cover it with a piece of stiff paper. When the class begins, proceed:

c) Lay down 4 differently shaped leaves, thus: oo oo. Note the gap between the pairs.

d) Pointing to the leaves, ask, “How many leaves are there?” Answer, “There are 4 leaves.”

e) Acknowledging the answer, point to the first pair of oo oo leaves and say, “2 leaves.” Then pointing to the second pair, again say, “2 leaves.” After a pause of a couple of seconds with a smile say, “2 leaves and …..” with twinkle and a flourish uncover the 4 () leaves and finish your sentence with “ ….. 2 leaves put together are 4 leaves.” : oo oo ()()()().

f) Continue with the tables and as in e. uncovering the appropriate () leaves thus:

oo oo	()()()()
oo oo oo	()()()()()()
oo oo oo oo	()()()()()()()()
oo oo oo oo oo	()()()()()()()()()()
oo oo oo oo oo oo	()()()()()()()()()()()()

g) Repeat c. d. and e. three times then either say you forgot or with an appropriate story draw the infant–child’s attention to the fact that the pattern looks untidy because it does not start as it should i.e. 00 ()() and insert these leaves at the very top.

h) A little dishonesty is suggested to enable the infant-child to conform to the way tables are usually taught. “Two ones are two” is the first assertion but it is a tautological assertion, perhaps not to the mathematician but certainly to the infant–child.

i) When the children have passed the test for 2 X 5 or 6 = 10 or 12, proceed to the end of the table, 2 X 10 = 20.

j) As the infant–child’s number vocabulary increases, it will be possible to move to the higher tables and it is possible that over a couple of years the little girl may be fluent with the 20 times table.

Test

Seat the infant–child to be tested in front of 10 others also sitting on chairs but with their feet planted 30cm apart. The teacher’s, “2 legs are…” is the cue for the infant-child at the left end of the queue to bring her feet together and for the examinee to complete the teacher’s sentence, loudly with, “... 2 legs.” When the teacher says, “2 legs and 2 legs are…” another pair of feet are brought together and the examinee yells, “4 legs” and so it goes.

Note: The multiplication of larger numbers will be covered later.

Activity 7

Note: There are 3 kinds of divisions, the one without a remainder, the one with a remainder and the one in which the remainder has been divided into as many equal parts as indicated by the divisor. Each question must have 3 answers unless the question has only one or asks for only one.

Objective

To experience the division of 16 by 3.

Materials

16 each of 6 different small objects such as toffees, stones, counters, marbles etc., 3 containers of glue, 30 small slips of paper and 3 sets of 10 coloured pencils.

[Suppose you have some toffees which you wish 3 infant-children to share equally. You can do this in one of two ways: Either (Activity 7) give one toffee to each girl in turn until 2 or less are left, then cut each of the remaining toffees into as many equal parts as there are children and give one part to each child; in this case all the toffees are shared and there is no remainder OR (Activity 8) make groups of 3 toffees, one for each girl and when you are left with to 2 or less, keep them aside as a remainder.]

Method

a) Place 16 toffees in a row.

b) Invite 3 infant–children to play a game in which all the toffees will be shared equally.

c) Draw a circle or a square or a triangle on 3 pieces of paper, place them upside down and ask the 3 infant–children to pick one each.

d) Give each little girl 6 slips and a set of 6 coloured pencils; ask her to draw on each slip with a differently coloured pencil the shape she picked.

e) Explain 'clockwise'.

f) Ask the girls, in clockwise order, to place on the toffees one slip, each girl in turn, the colours being first red (R) then blue (B), green (G), yellow (Y), orange (O) and pink (P) thus:

Shape: C = circle, S = square,
T = triangle

C S T C S T C S T C S T C S T C.

Colour:R R B B BGGG Y Y YOOO P

g) Ask the girls to pick up a toffee and its slip from the left of the row, each girl in turn, and place it in front of herself until the row is exhausted.

h) They will end up with

Girl:	1	2	3
Shape:	C C C C C C	S S S S S	T T T T T
Colour:	R B G Y O P	R BGY O	R BGYO

i) When asked, the girls will say that the stones were not shared equally.

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PART III

Interview with Jasbir Bahiya

Q What should be the aim of a teacher teaching mathematics?

Jasbir Bhaiya: The purpose of a teacher is not to teach. Sri Aurobindo says "The first principle of ture teaching is that nothing can be taught. The teacher is not an instructor or task-master, he is a helper and a guide. His business is to suggest and not to impose." The teacher will always keep himself open to pick up the signals that emerge from a child and maintain the interest of the child in learning. He will not force the child to learn what he has planned for the child. If the teacher is open enough and creates an environment according to the need and interest of the individual child then the child will learn naturally.

> Mathematics is a language that children begin to understand

Q How will the teacher get ideas from a child?

JB: The teacher will observe the area of interest and disinterest, the area and method in which the child is confident; in which he is not confident or struggling and withdrawing himself. He will identify the precise problem of the child and try to understand what the child is asking for. So the teacher should be a vigilant observer, without blaming the child for what he is not doing.

So the planning for the child should happen on the spot.

Pre-planning biases the observation. It does not give time to the children to explore by themselves. Pre-planning education becomes teacher-centred or teaching-centred, not learning-centred and child-centred.

Q How does a child learn?

JB: You see that a small child is not taught who the mother is. And what a mother is. He is not taught what water is or what red means. He learns from Nature, from the surroundings with all his senses — hearing, seeing, touching, and smelling. He also learns by tasting. He uses all the senses for his learning. Interestingly, when a child learns a language he speaks a little differently from our normal style of speaking but we all enjoy his speaking style and we too become children, encouraging him by mimicking his language. We do not try to correct him, and slowly he learns the correct way by listening to others. This same approach is relevant for learning the language of mathematics too. However, in general, we tend to correct the child from the beginning, instead of letting him learn by observing. We give instructions again and again, pointing out mistakes.

The numbers 1, 2, 3… do not make any sense to him. He enjoys speaking one, two, three, and so on. For him sequence or order has no meaning, because the logical faculty has not developed. We teachers and parents do not understand the biological growth of a child. We think that we will dump everything on the child and he will remember it like a parrot. But this is not the actual way of learning and we need to realise that.

We all believe mathematics is a subject that emerges from the mind. So we always treat it from the mind. But really, for a child it is not from the mind, it is from the heart.

Try this exercise with a child. Tell him, “I had three toffees in my pocket and two fell down on the way while I was running. Now how many do you think remain in my pocket?”

Do this exercise by putting three toffees in a bag and taking two away. He will see one remaining in the bag. This is the problem that the child relates to everyday in life. So it does not matter if the child does not remember 3-2=1 since it has no meaning for him. If you show it again with different objects by telling different stories he will be able to relate. So the point is that a small child does not remember anything that he cannot visualise. His memory does not develop at the early stage. So always repeat the exercise with different objects and slowly the child will be able to make the relation with numbers.

Q How do we go about this…

JB: We always think the child is an empty vessel that we need to fill up. But real education is other way round. Everything is inside a child and we need to help him to unfold like a flower. Our job is not to open the petals of a bud but to give it the environment and the bud will open itself. We do not correct the language of a child when he learns. We speak in front of him and he picks up from the surroundings. We need to maintain the same when a child is learning mathematics. There is no need to teach him concepts or force him to remember what is correct. We always need to show the child by doing different exercises. The child will do the exercise and get the correct answer out of it.

> ….open to pickup the signals that emerge from a child

For example, when we teach a child 2+3=5. the child just remembers without understanding the meaning. For him, “2+3=5” is a figure, a pattern like any other picture. So we need to show him by using different objects, creating different situations again and again. The child will conceptualise by himself looking at what is happening.

For him:
2 toffees merge with 3 toffees
2 balls merge with 3 balls
2 pencils merge with 3 pencils

From a multitude of such experiences, he will draw the conclusion 2+3=5.

So we need to be aware of what is going on in his mind while giving him objects.

When a child learns numbers, he does not distinguish between the words 2 and toffees. For him 2 toffees is one word, 2 apples is one word. If he is asked what two is, he is not able to separate 2 from apples. If you separate 2 and apples he will get confused. After seeing varieties of two objects, he then realises what two means. So we need to be very alert while doing mathematics with small children.

The learning of mathematics does not start with abstract symbols. It starts with real life situations. So do not worry about teaching symbols first. Do as many exercises as you can do with stories, giving real life situations with real objects. Slowly shift to pretend objects for bigger numbers. Then the next shift is to the mind. While focusing on the mind, a lot of mental exercises can be done with the children before going to written mathematics.

Do not hurry to state the logic behind any concept. Give sufficient freedom and time to a child to discover the logic. Don't get him to remember the logic. That is the true learning of mathematics. When we give any time to a child to play, we think of that as wasting time. So we rush to tell the answer. This is a great mistake in the teaching-learning process. A child learns when he plays.

The job of a teacher is not to tell the answer but to show how to get the answer. He has to encourage and create enthusiasm to find the answer. Let the child make mistakes several times. Ask him to find his mistake and guide him. Never point out the mistake. There is great learning in making mistakes and discovering it oneself. If we are to give the answer to the child then the child never learns how to correct himself. How much can you correct throughout his life? A person needs to know how to find the mistake and walk on the correct path. So the work of a teacher is to show the way to find one's own mistake and rectify it.

Mathematics always gives the truth. It has rigidity in it; one cannot go beyond the discipline. For example, 3+2 is 5, not 6. Any place you will go it is the same. That is why mathematics is called a subject of truthfulness.

Acknowledgement

I acknowledge my debt to the following adults and children who have made this essay possible:

- My son Ranvir because he was the only infant around to teach me, when I started pottering with the interplay of adults, infants and intuition and now because he has gently dressed my thoughts in form and language which makes this essay comprehensible.
- Neeltje and Matthijs who accepted me at mirambika, which gave me the opportunity to interact with children.
- Vijay Varma and his School Mathematics Project Team for giving me my first opportunity to discuss Initial Mathematics with teachers.
- The children of mirambika who taught me so much over 20 years, gave me so much more and most importantly bestowed their trust upon me.

Thank you Ranvir, Vijay, Neeltje, Matthijs and most importantly mirambika and her children.

By Jasbir Singh Malik

How do we proceed?

Now, with some basic understanding of Initial Mathematics, its aim and approach, and its role in the integral development of the being, the child, how do we proceed from here?

Integral education, i.e. education guided by the soul, follows Sri Aurobindo's three principles of true teaching:

- Nothing can be taught
- Child's mind has to be consulted in its own growth.
- Work from the near to the far

Integral education rests on the fact that all knowledge is within the child. It is therefore, essential that the facilitator is convinced about this, either through some years of practice and or through a rigorous introspection. The facilitator, with this conviction, then builds a trust in the child in the facilitator and in her own abilities. The facilitator never puts down a child, rather, every attempt is made to draw out the understanding from the child. The outer environment is created when the facilitator sets up a stimulating space around the child and arouses her natural curiosity and interest to learn. The second principle - the child's mind has to be consulted in its own growth – calls for an emphasis on the process more than the answer. The process here refers to the way in which the child understands. The facilitator has to observe and discern that and respect the unique intuitive way in which the child solves problems, and to find a way to nurture this. The third principle is to work from the near to far. This requires facilitators to communicate with the child using examples or objects familiar and most important to the child. It requires the facilitator to be familiar with the world of the child. It is only when the child gains in confidence within her own world that she is led to worlds other than hers.

Keeping in mind the nature of integral education, where the aim is to nurture the child as an independent learner, space and time is afforded to the child to explore her relationship with numbers and from thence, the learning of mathematics in her own way.

In the group, each facilitator prepares a tentative plan for the children's development guided by their individual and collective abilities, interests and needs and not by a fixed curriculum. Needless to mention, the plan also varies from facilitator to facilitator depending on individual strengths and quests. (Appendix - Curriculum for various age groups prepared in mirambika for a year.)

Mathematics learning, as much as possible, is made interesting, and relevant to children's life experiences. The following are various ways of introducing numbers to the children through the play-way approach:

1. learning through practical and concrete real-life experiences.
2. observing the patterns in nature.
3. through story telling.
4. making friendship with numbers through music.
5. problem solving through imagining situations.
6. playing indoor and outdoor number related games.
7. learning calculation through abacus.
8. learning by doing.
9. using the activities like paper folding, rangoli making.
10. making riddles, poems and stories.
11. doing mathematics related project work.

Some practical considerations

The familiarization with numbers starts with objects, not numerals. In this way, the children are guided to develop a relationship with numbers in a step-by-step manner. Firstly, they establish their relationship with numbers through concrete objects followed by pictorial representations. They then progress on to mental calculations with small numbers followed by a shift to abacus, which serves as a bridge between the concrete and imagined. It is only after these are established that children become ready for abstract mathematics.

Another important consideration when carrying out counting with children is to work with similar objects at a time, in order to avoid confusion. For example, they could count their friends, bottles and bags, one type at a time but not a mix of these. Mathematically, teachers or facilitators could do with using well-defined sets starting with familiar objects.

Nature abounds with beauty and has ample resources for our purposeful utilization. Children have a natural affinity to Nature. When left alone in Nature, they remain one with it, and true learning begins in this state. They observe the colours of nature, touch the patterns close to them, hear the sounds all around them, feeling the rich essence of nature deeply through all their senses. The colourful wings of a butterfly, the beauty of a flower, the pattern of petals, the softness of leaves, the hardness of stones – all these attract their attention. They are keen collectors, identifying with their discoveries and making imaginative associations. Each child is attracted to different elements in nature and relates with patterns in a unique way. To one child, a leaf can be an aeroplane, to another, a patterned seed cover can be the bogey of a train, and to yet another, a bent stick can be the head of a giraffe. These associations are the foundation for the logical connections of later years.

Here are some experiences gleaned from mirambika, illustrating the above discussion.

On one occasion, a child in the Blue Group (age group 4+) found a stick in the outdoors and started measuring himself. He put the stick in front of him touching his body and said, "ये तो मेरे जितना है"। Then he put it in front of didi and said, "पर ये दीदी से छोटा है". His eyes quickly found another stick lying on the ground and he said, "ये तो दीदी से भी बड़ा है". Then he found another stick and said, "अरे ! ये तो दीदी के बराबर है". He looked satisfied but the learning did not end there. It created an interest in all the children to collect various things from Nature and use them for measurement too. In this way, children could learn the concept of quantifications, such as 'big', 'bigger', 'biggest', 'small', 'smaller', 'smallest', 'tall', 'taller' and 'tallest'.

One day, all the children decided to measure the depth of the fish pond by saying "हम fish pond में कितना पानी है देख रहे हैं?". When they were asked, "कितना है?" They replied, "हमारा knee तक पानी है". After a few seconds one of them again said, "अगर मैं गिर जाऊँ तो मेरा कुछ नहीं होगा". It's amazing how far the child's mind can project and reach.

Mathematics is nothing but life itself and it helps the child to learn to live the life. Hence, it cannot be confined to the classroom. The learning and mastery of mathematical thinking by the child calls for the teacher or facilitator to be vigilant in creating opportunities in the course of the day, in any space, while spending time with the child, and creating various learning opportunities for the child. However, this should be more a spontaneous process rather than regimental. Learning can happen spontaneously naturally in the playground, while eating, while decorating a space, while exploring in nature, and while doing just about anything. This is the wonder of mathematics.

CHAPTER 04

Pre-number Concepts

Before introducing numbers to a child, some preliminary concepts must be introduced. We call these pre-number concepts.

Most of the time we are happy to say, “My child can count from 1 till 20.” If you ask a 4-year-old, she can recite numbers from 1 to 20 in the correct sequence. When you ask her to bring 7 pebbles from a pile of pebbles, the child will recite till 7 but bring you more or less than 7. Have you experienced this?

Why does the child do so? Can we really say that the child knows counting? What we observed is that the child has memorised the name of the numbers in sequence. However while counting she is saying a number without touching a pebble, or counting one pebble twice, or keeps saying the sequence even after the pebbles have been exhausted.

Another instance: Keep some beads in a row and ask a 4-year-old to give you 2 beads. The child may start counting one, two…and give you the second bead. Have you experienced this?

In this case, she understands two as the second object and not as quantity. Hence before counting we need to develop some concepts like classifying, ordering and one-to-one correspondence.

This stage of learning is known as the sensory learning stage. In this period,the development of senses in a child is the most important. So the activity planned for this level of children needs to be based on the five senses – hearing, sight, smell, taste and touch.

From early childhood the child is able to distinguish between family members, toys and colours. While playing with toys, he can separate one from others. When he does not see a family member for a while, he starts searching for him or her. This means that the child has intuitively started distinguishing, classifying and matching. It is, therefore, quite wrong to say that mathematics starts when we start teaching 1, 2, 3. The mathematical process started in the child naturally, from childhood. It is god gifted. As diyas and parents, we merely guide children, show them the path and make them aware about the terms which we use in everyday life.

Now we will discuss how the pre-number concepts can be introduced to small children through different interesting ways.

1. Classification or grouping

2. Ordering or sequencing

3. Pairing or one-to-one correspondence

1. Classification

Classification happens in two steps. One is matching and another is sorting. Identifying the common properties or common characteristics (shape, size, weight, number, or material, for example) of objects is called **matching.** Putting objects of the same category together is called **sorting**. While doing matching another action takes place and that is **comparison**. This is the ability to discriminate between similar and different things based on attributes such as colour, texture, shape, weight, size, or any other parameter. The vocabulary that we need to develop in a child through this concept includes the following terms:

- long
- short
- as long as
- longer than
- shorter than
- round
- thinner
- same

Activity 3: Children collect different types of leaves. These are sorted out, and similar sets of leaves are grouped and placed in separate places (boxes, bags…)

Activity 4: Children collect different types of flowers and sort them according to the number of petals.

Children assemble items they have collected from nature for classification

Here are some activities practiced with children of age group 4-5.

Activity 1: Children collect different things from Nature and sort them according to colour, size, shape or length.

Activity 2: Children collect different things from Nature and sort them according to properties like floating, sinking and flying.

Activity 5: In the group, the diya asks the children to allot separate places for different materials like crayons, pencils, board and eraser, and place all such objects in their respective places.

Activity 6: In the dining hall, children are asked to arrange different things like plate, spoon, bowl and glass in their respective places.

Activity 7: In the group, the diya asks the children questions like – “Those who are wearing red T-shirts, please stand up" Or "Those who are wearing half-pants, please stand up."

Activity 8: The children are given some picture cards like flowers, plants, animals and vehicles. Concrete objects like toys of flower, plant, animal and vehicles are kept in front of them. They are asked to match the picture cards with the objects.

Activity 9: The children are given a set of different shapes and asked to sort them by keeping all the squares in one place, all the triangles in one place, all the diamonds in one place, etc.

2. Ordering and Seriation

Ordering means arranging a set of objects in a sequence according to some rule. This arrangement could be on the basis of shape, size, colour, texture and other attributes.

Seriation is a particular type of ordering in which the objects are arranged in the increasing or decreasing order of some attribute like length, size, weight, volume, count and so on.

Note: While preparing the activity keep your children's abilities in mind. Activity must start with the simple and progress to the complex, increasing according to the level of the child. While starting out, seriating more than three objects may be difficult for young children.

Activity 1: Collect flowers or leaves, preferably with the children. Ask them to make some repeating pattern with the flowers or leaves in one line. If they are not able to do so independently, help them to start and then they can follow.

For example,

1. (leaves) brown, green, brown, green…

2. (leaves) small, big, small, big…

3. (flowers) white, yellow, white, yellow...

4. (leaves) thin, broad, thin, broad…

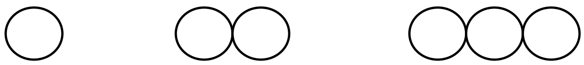
Making patterns with flowers and leaves collected from nature

Activity 2: With different coloured beads or colour crayons, show some sequence. Ask the child to extend this sequence.

Activity 3: Give some sticks of different length and ask the child to arrange these from smaller to bigger or in reverse order.

Activity 4: Give different sized triangles (or rectangles or circles) and ask the children to arrange them in increasing or decreasing order of size.

Activity 5: Show the children an arrangement of beads in increasing order of numbers and ask them to extend.



Note: It is important to keep up a parallel conversation during the seriation activity. This will help introduce the child to different mathematical terms like small, smaller, smallest, long, longer, longest, large, larger, largest, big, bigger, biggest etc. Also, using words like last, first, before and after helps children to understand the concept.

3. One-to-one correspondence

This concept helps the child to develop hand-eye co-ordination. It harmonises the child's thinking, doing and speaking ability. This makes it easy for the child to relate to numbers with objects/things/materials/people etc.

Here are some activities that help children to practise one-to-one correspondence

Activity 1: Some bottles are kept in a line, and the children are asked to put caps on them.

Activity 2: Some dolls are arranged in a row and the children are asked to keep one sweet for each doll using beads as sweets.

Activity 3: Ask the child to keep a pretend toffee in a row for each child of the group.

Activity 4: The children could arrange one table with one chair and count them as "1 chair - 1 table." While they count, it is important to touch the respective chair or table.

Activity 5: Children collect flowers and feathers and make them friends (one feather will make friends with one flower) and arrange the pairs in a row.

One to one correspondence

Activity 6: In the dining hall children can arrange plates and bowls, placing, for example, one bowl next to one plate.

Activity 7: Given some shapes, children are asked to arrange one square with one triangle, and so on.

Activity 8: Give identical shapes (but in different colours) to the children and ask them to make friendship between identical shapes of two different colours like blue with red or green with yellow (any two colours)

Children love to engage in these activities especially when they themselves collect things from nature. They spontaneously make different patterns and designs on the floor using these things and sometimes narrate stories about their creations. This natural exploration helps them to understand the pre-number concepts in an intuitive way.

Sometimes, observing the children's interest, pattern and design are taken up as a project for five year olds.

Making patterns and designs with things collected from nature

Making intricate patterns and designs with various things

CHAPTER 05

Counting and Estimation

Before a child starts to learn counting, she should go through the process of all the pre-number concepts which are essential to the process of counting. Here we are going to discuss all the steps in counting.

Steps of counting

Imagine that you are asked to count all the red coloured laddoos from a pile of different coloured laddoos.

First you will match all the red coloured laddoos.

You will pick them out and sort the laddoos in two sets: red colour and non-red colour laddoos.

Now you will arrange the laddoos in an appropriate order for counting, such that no laddoo overlaps another or is hidden from view. You will order them in a row or in a pattern so that you do not leave any out or count any twice.

Now, you have an ordered row of laddoos to count. While counting, you will call out the numbers one, two, three … and for each number, touch a new laddoo, without leaving out any or counting any twice. (You will end up saying as many number names as there are laddoos.)

Suppose you say ten while touching the last object, you conclude that the number of laddoos is ten. This set of 10 laddoos must now be grouped in a visible manner, such as putting them onto a plate or looping a piece of string all around them.

(Even if you were to do this at a glance, you would still be following all these steps. The difference is that you would be doing them so quickly that you would not be conscious of all the separate steps.)

Beginning of Counting

Counting starts at an early stage when the child starts gathering toys, collecting materials with his playmates, sharing sweets or toffees on his birthday, collecting flowers, twigs and leaves from the garden etc. Normally it starts with concrete objects that are very familiar to the child. For example, while introducing two, show the child two leaves, two hands, two balls, two pencils and two toys or clap two times. Ask the child to clap with you, jump and hop with you two times. One can introduce a variety of experiences. In this way, all the small numbers till 10 can be introduced to the child. Counting by touching the objects is important to develop the sense of touch, essential for integral growth. The child will touch and feel the individual objects and associate the objects with the count.

‘Fire on the Mountain’ is a good game to not just experience different numbers, but also to develop alertness.

Fire On The Mountain	
No. of players	Any number, though at least 10 are required to have a meaningful game

All the children will run in a circle saying, “fire on the mountain, run, run, run” repeatedly. Didi will call any number like four, five, three etc. When the children hear the call, they will collect into groups of the called number. Those who are not able to be in group may lose one point.

It is best if the numbers are not always used in a conventional order (1, 2, 3…). In a previous chapter, we learned how some children simply memorise the numbers in the correct order, without understanding the value. If the numbers are always presented in the same fixed order, the child may not explore the numbers freely. It is best to present the numbers that already exist in the child’s world. One can ask questions like:

- How many hands do you have?
- How many wheels does a tricycle have?
- How many red seeds are in the basket?
- How many flowers did you collect?
- How many noses do you have?

Such an approach will help the child explore and build a relationship with numbers instead of simply memorising the number sequence.

One can interact with a child as a playmate, not as a teacher, playing with toys and different objects. Slowly start making some designs with similar kinds of objects. Ask them how many similar things have been used in the design. Playing with different objects, the child will gradually become familiar with the numbers from 1 to 20.

Counting by seeing patterns

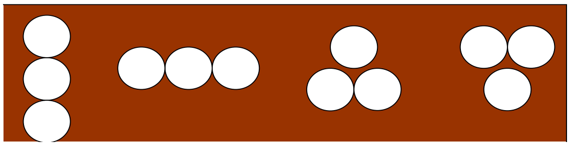
Once the child is familiar with counting objects by touching and separating one from the other, we are ready to introduce counting by seeing.

Activity 1: Changing beads

Aim: To develop observation

Starting with a fixed number of beads, the diya makes a pattern and asks the child to count. Then the diya changes the pattern in front of the child and asks the child to count again without touching. She then repeats this kind of activity with the child with different number of beads till the child is able to count by visually looking at the patterns.

Example: Three beads can be arranged in different patterns like



Activity 2: Creating patterns

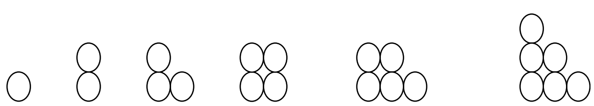
Aim: To develop creativity with counting.

i. Take a number of beads and ask the child to group them and make different patterns. Then ask the child to change the pattern. Note that the child is not required to count during this exercise, which is more to build familiarity with pattern-making and become aware of numbers.

ii. Give a fixed number of beads to the child and ask him to arrange them in a pattern. Then give the child another set of the same number and ask him to form a different pattern, beside the previous one but not touching it. This gives the child the opportunity to compare the two sets and, perhaps, intuitively recognise the similarity in terms of the count. More patterns can be added in this way, creating a wider set for comparing and noticing similarity.

iii. Invent simple games and play them. For example, make pairs of children. One partner will show a bead pattern for a short time, just a glance. The other partner then has to tell the number of beads. Then they change places.

iv. Ask the child to arrange beads in different patterns according to increasing or decreasing order.



Do not introduce these terms to the child. Instead, give guidance such as 'this should be more than that' or 'this can have less beads than that'.

Counting with proper Sequence

When the children are familiar with the numbers and are able to arrange them in ascending or descending order, show them the 10-beads-string. [1]Show it in such a way that the child can see clearly. Start with all the beads on one side. Then move a single bead to the other side and ask the child how many beads you have moved. Continue, one bead by one, till 10, all the while asking the child, "How many now?" After the child becomes familiar with this, introduce reverse counting, asking how many beads remain after moving each bead.

While introducing counting, it is important to count with the child and move one object while saying "one", move two objects while saying "two", as well as asking the child to touch each object. A lot of practice is essential in this regard. It is necessary to use a variety of objects for one concept.

1**Beads-string** can be made by collecting beads and threading them on a strong string. Both ends of the string should be knotted separately. Colours can be used to show groupings. Extra strings should be kept to tie the string while presenting to the class. *(Readymade kits are available from Jodo Gyan.)*

It obviously helps to hold the interest of the child. More importantly, the child will associate numbers with different objects, not any one object. In the same way, for bigger numbers, one can use **Beads-string** of 20, 50 or 100.

Some activities for counting "Beads-string"

Before introducing any Beads-string to the children, ask them to guess the number of beads. The children will come up with many answers. Then ask them to count one by one and check their guesses.

Activity 1

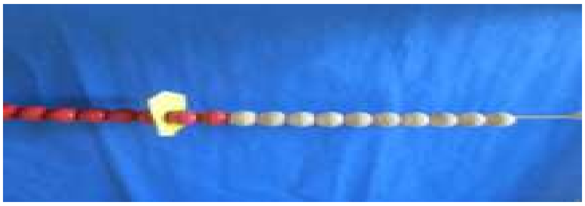
Children are asked to count the beads one by one.

Activity 2

A marker[2] is put at any place on the string and the children are asked to count till the marker and tell the number.

Activity 3

The children are asked to put the marker on the string acording to a given number.



Note: *Observe how the child counts. Initially he is encouraged to count one by one. Later, try to develop the practice of recognising the colour pattern of the beads and counting in groups of 10.*

Some other activities on counting

Activity 1

Counting with sticks, stones, leaves etc.

Aim: To develop counting ability

Children are taken on a nature walk and told to collect some materials. Then they are asked to count what they have collected.

2 Markers can be any material which, cut into small squares with a slot, can hold their place on the beads-string.

To make it more interesting, they could count the numbers of each type, such as leaf, twig, stone, combinations of these, and also the total objects. This will help to associate the numbers with different objects, reinforcing the number concept.

Activity 2

Counting with one-to-one correspondence.

Aim: To develop eye-hand co-ordination and build association between the objects and the count.

The children are given a certain number of stones or beads. Then they count aloud while touching each object.

Activity 3

Counting from a basket

Aim: To develop alertness and thinking in counting and also to know the quantity of the number.

Children sit in a circle. A basket with 55 beads (or pebbles) is placed in the middle. Start with any child, who takes one bead and says "one". The next child takes two beads and says "two". The game proceeds till all the beads have been removed.

You may find that after the 10^{th} child takes 10 beads, a few beads are still left. This would happen if one or more of the children took fewer than they counted. Ask the children to quickly count the remaining beads.

Similarly, if someone takes more beads than the count, the last child will not have 10 beads left. Ask how many more are needed to get 10. Some children may intuitively do the subtraction, but one should not expect this.

Activity 4

Making different shapes using a fixed number of objects.

Aim: To develop the faculty of creativity and quick thinking. Optionally, to practise estimation of larger numbers.

The child is given 10-20 beads and asked to make a tree, a flower, a circle or a triangle etc. using these beads. Then the child makes things according to his imagination.

As an additional activity, two children can make a shape each, then take turns at guessing the number of beads used by the other, after a quick glance.

Activity 5

Guess the numbers

Aim: To develop estimation ability

The diya shows a few sticks or stones and then hides them. Each child guesses the number. After all the children have guessed, it will be counted before them to check who is correct or closer to the number.

Activity 6

Dot card game

Aim: Observing the number patterns and counting

Prepare a set of 20 cards with 1-20 dots respectively. (A sample design is shown here, but you may create your own patterns.) The cards are shown to the children, who count the dots and speak their answer.

Note: *While counting objects, encourage the child to make two sub-groups – objects already counted and those yet to be counted.*

Backward counting

Backward counting has an important role in faculty development as well as mathematical operation. The children need to develop the ability of backward counting along with forward counting. There are a few activities on backward counting which we practise with children.

Aim: To develop thinking, concentration and observation.

Activity 1

Children are given 10 beads at a time and asked to arrange the beads in a line. Then they are asked to take out one bead from the line of beads and tell how many remain. Again take out one bead from the line and ask how many remain. This goes on till the last bead. When this bead is removed, the children may say that there are no beads left. Here you may introduce the term 'zero'.

Activity 2

Use Bead-string of size 10, 20, 30, etc. and start counting backward, subtracting one bead at a time from the whole.

Note: For older children, use a 100-bead string and count backward from 100.

Skip counting

Aim: To develop memory and gently introduce the pattern for multiplication tables.

Activity 1

The children are asked to keep 2 pebbles and say 2, and then they are asked to put 2 more with previous two and count then say 4. Like this each time they are asked to put 2 more pebbles with previous one and count the total pebbles and say the number.

Note: For skip counting 3, 4 and 5, we will follow the same process, using multiples of 3, 4, and 5.

Activity 2

Sitting in a circle, the diya gives some pebbles or beads to each child. The first child says “2”, while putting two beads in the pot, then passes the pot to the next child. The next child says “4”, adding another two beads. This continues, with the count increasing 6, 8… For 3, 4 and 5 skip counting follow the same process.

Activity 3

(*This activity uses what we call ‘number catchers’, bamboo clips made to hold 2 or 3 or 4 beads.)*

Tie each end of a 100-beads string to a support so the children can see the entire string. Ask the child to count 2 beads aloud, use the 2-number catcher[3] to bracket the beads, slide them to the other end, then bring back the catcher for the next 2 beads. Repeat the same till all the beads have been moved, counting along while moving the beads (2, 4, 6...).

The same procedure can be followed for skip counting 3, 4, 5, etc.

3 Catchers can be made from cardboard,bamboo,or any suitable material. If using cardboard, simply make a cuboidal box, open on one side, so the ‘caught’ beads are visible. If using bamboo, cut a suitably sized strip, with two brackets glued at the correct distance to ‘catch’ the required number of beads, (5 beads catcher shown in the picture above.)

Activity 4

Sit in a circle and start counting mentally. First child will start with 2 and say it aloud, next calls out “4”, the next “6”, and so on. (This can be done only after lots of practice with objects and beads–string). Similarly skip counting with 3, 4, 5 …etc. can be done according to the age of the child.

Activity 5

Playing ‘Bingo’

Aim: To develop alertness and practice skip counting

All the children sit in a circle. First we decide the Bingo number 3, 4, 5 etc. If the Bingo number is 3, the first child will call 1, the second child will call 2… but the third child, instead of calling 3, says ‘Bingo’! Each time a multiple of the Bingo number comes up, the child should call Bingo instead of the number. The child who forgets to do so loses a point.

Activity 6

After a lot of regular skip counting, we can introduce generic skip counting (starting with any number, not just the skip number). For example, generic skip counting with 2 need not start with 2, but can start with 1 (1, 3, 5, 7…) or 5 (5, 7, 9, 11...). Similarly skip counting of 3, 4, 5, can be practiced starting with any number. This can be done with the Beads-string or mentally.

Breaking of Numbers

Aim: Breaking of number has an important role not only in mathematics but also in faculty development. It helps widen creative thinking and relating one number with other numbers. That means seeing a number with reference to other numbers.

A. Breaking the number in two parts

When the children are familiar with counting till 10 we play some games with beads.

Activity 1

Aim: To develop the relationship of one number with other numbers

A fixed number is given to the child (or chosen by the child.) The child is then given several beads. He then has to take the given number, and separate it into two parts. He then takes another set of beads of the same number, and separates this new set into two parts, but in a *different* way from the first.

Example:

The child chooses the number 5. He then creates one set with the 5 beads split into 1 and 4 beads. Then another five beads, which he now splits into 2 and 3 beads.

Activity 2

Aim: To develop the ability of breaking a number mentally, practice mental addition, and have fun.

Show a fixed number of beads to the children and ask them to count these.

Separate them into your two hands, taking care that the children do not see how many you picked up in each hand. Now ask them to guess the number in each hand. Children will usually give different answers and then can check who is right. This game creates a lot of enthusiasm and curiosity among the children. Once they get the idea, the children can form groups of two or more and play this among themselves.

Activity 3

Make a three mouthed pipe with chart paper or wood. Keep one mouth up and the other two down. Show a fixed number of beads to the children and let them count. Drop the beads into the upper mouth. Let the children count how many come out of each of the lower mouths. Repeat this, but now, before dropping the beads in, ask the children how many will emerge from each mouth. Then drop the beads in and have them check their predictions.

The element of chance makes this an interesting game for the children.

Number Card Game	
No. of players	4
	10 cards with numbers 1-10, written using words, not numerals (4 such sets, totalling 40 cards).

Aim: To break a number mentally, practice mental addition, and have fun.

Before starting the game, decide a 'break number', from 6 to 15, both included. (In the following pictures, our break number was 'ten.) The object of the game is to get two pairs of matching cards , i.e. two pairs, each adding up to the 'break number'.

Shuffle the cards well and distribute four cards to each player. Keep the rest of the cards in a pile facing down. Turn the top card face-up and place it at the centre. Turn the top card face-up and place it at the centre.

The first player checks the cards in his hand and tries to find a match with the card at the centre (say, if the card at the centre is a ′two′ and the ‘break number’ is ‘ten’, he would hope that he is holding a ‘eight’).

If he does have a matching card then he shows it to the other players, takes the card from the centre. In our example, the player shows the eight and picks up the two.

He now discards one of his other cards (‘six’ in this example). This goes on top of the face-up pile and the next player must now match this card.

The second player now needs a ‘four’ to complete a pair (10=6+4). If he does not have a ‘four’, he can pick the top card from the face-down pile. If it is a ‘four’, that’s good luck for him, and he can proceed as explained above.

If it is not, he cannot pick up the ‘six’. He is now holding 5 cards, and must discard one. In our example, he discards an ‘eight’. The turn now passes to the next player, who now needs to match the ‘eight’ (with a ‘two’), as the ‘six’ has been covered and has gone out of play.

Play proceeds in this way till one player has a full set, i.e. two pairs of matched cards.

If all the cards in the pile have been exhausted, leave the top card on the table, shuffle the rest and turn the pile face-down.

Note: For children unfamiliar with reading, dot cards may be used instead.

B. Breaking a number into many parts

After a lot of practice with breaking a number into two parts we try to break the number into more than two parts. The children use their thinking ability and try to explore in many ways.

e.g. Children are asked to arrange 5 stones (or beads, or any other objects) in as many ways as they can.

Guide the children to make aesthetic arrangements, so that they remember the patterns.

Here is a complete list for the number 5.

•	••••			
••	•••			
•••	••			
••••	•			
•	•	•••		
•	••	••		
•	•••	•		
••	•	••		
••	••	•		
•••	•	•		
•	•	•	••	
•	•	••	•	
•	••	•	•	
••	•	•	•	
•	•	•	•	•

C. Splitting numbers into tens and ones

Activity

Grouping objects into tens and ones.

Aim: To develop the faculty of observation, thinking and alertness. To understand the pattern of tens and ones.

Children are given sticks and bundle them into groups of 10, each secured with a rubber band. They do this for 10, 20, 30, and larger round numbers. For each number, ask them how many bundles of ten they have. Then proceed to numbers such as 12, 15, 24, 36 and so on. For each, ask them how many bundles of ten they have, and how many loose sticks.

(Encourage them to show bundles on the left side and open sticks on the right.)

For example the number 24 will be shown by two bundles of 10 sticks on the left and four open sticks on the right.

After a lot of practice with stick bundles, they can try to break the number into tens and ones mentally. Like 23 contains 2 tens and 3 ones.

This concrete experience with the stick bundles helps the children to understand addition and subtraction before they proceed on with written mathematics.

Sweet Mother, There are some things which are good for my progress but seem to me very uninteresting. For example, mathematics is a good subject but it does not appeal to me. Please tell me, how can I take interest in the things to which I am not drawn?

There are a lot of things that we need to know, not because we find them specially interesting but because they are useful and even indispensable; mathematics is one of them. It is only when we have a strong background of knowledge that we can face life successfully.

- The Mother

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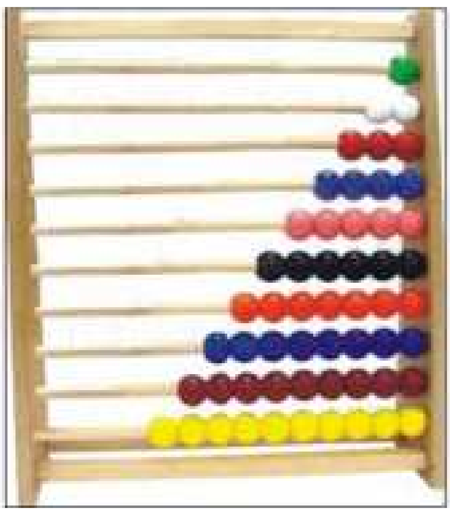
CHAPTER 06

Abacus

For a long time, people have used the abacus as a tool for calculation. Initially, it was made by keeping beads in a line on the floor. Over time, the abacus began to be made by threading beads on a string, and the string was tightened between two sides of a frame.

The **Slate Abacus** has 10 lines and the lines contain one to ten beads, respectively. This type of abacus is used in primary schools to learn counting from 1 to 10.



For larger numbers, another abacus is used. In this a base supports three or more poles which are labeled as ones, tens, hundreds and so on. Numbers are shown by putting appropriate number of beads on the respective poles.

(Here is an innovative construction of the abacus using an old rubber slipper and pencils.)

Brain Juggler: Abacus in mirambika

The abacus we use in mirambika is different from the abacus described above. We call it a brain juggler, for it can make the mind more supple, flexible and quick in the world of numbers. It is challenging, yet fun for the user.

This abacus contains ten rows, each having ten beads. It is bounded in a rectangular wooden frame. It is designed to handle any number system from binary to decimal.

Though we introduce numbers through different number systems, there is no effort to *teach* children which is binary system and which is decimal. We never even mention the term 'number system'. Instead, we refer to a number in code 2, code 3, code 4 etc. The main aim of working with the abacus is to train the child's mind to do faster and larger calculations and to derive relations between the numbers. As the mind develops, children are able to handle bigger numbers mentally without pen and paper. This is an intermediary stage between calculation with concrete objects and mental mathematics as far as larger numbers are concerned.

Introducing the Abacus

Generally we introduce the abacus at the age of 6+. For the first few days, children play with the abacus, becoming familiar with it by counting the beads and making patterns with them.

Then we introduce its use with different codes.

How to hold: Hold the abacus in your left hand and keep your right hand free for manipulation. *(Left handers may exchange hands, but the abacus must be held in the same position.)* Start with all the beads on the absolute left side of the abacus. (The number will be shown on the right side.)

Code 10

There are 10 beads in each row. The value of 1 bead in the first row is one. The value of 1 bead in the second row is 10. It is the same as the total value of all beads in the first row. The value of 1 bead in the third row is the same as the total value of all the beads in the second row that is 100. In general, we can say that the value of each bead in a row is equal to the total value of all beads in the previous row. In other words, the value of each bead in a row is 10 times the value of a bead in the previous row. So the value of 1 bead in the first row is 1, in the second row is 10, in the third row is 100, and so on. Therefore the value of a single bead in each of the 10 rows is, respectively, 1, 10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000, 1000000000.

When children are familiar with the values of beads in each row, they can be asked to form different numbers by using the abacus. Children must be encouraged to use the minimum number of beads in a row. For example, to represent the number 134, one bead from the third row, 3 beads from the second row and 4 beads from the first are used. For number 23456, 2 beads from the fifth row, 3 beads from the fourth row, 4 beads from the third row, 5 beads from the second row and 6 beads from the first are used.

For practice, children could show different numbers on the abacus.

Code 2

Keep 2 beads in each row. The value of 1 bead in the first row is one. The valueof 1 bead in the second row is the same as the total value of all beads in the first row. The value of 1 bead in the third row is the same as the total value of all the beads in the second row. *In general, we can say that the value of each bead in a row is equal to the total value of all beads in the previous row.* In other words, the value of each bead in a row is 2 times the value of a bead in the previous row. So the value of 1 bead in the first row is 1, in the second row is 2, in the third row is 4, and so on. Therefore the value of a single bead in each of the 10 rows is, respectively, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512. When children are familiar with the values of beads in each row, they can be asked to form different numbers by using the abacus.

1
2
4
8
16
32
64
128
256
512

Always encourage them to use the minimum number of beads in a row. In case of code 2, not more than one bead can be shown in a particular row.

For example, to represent the number 3, use one bead from the second row and one bead from the first.

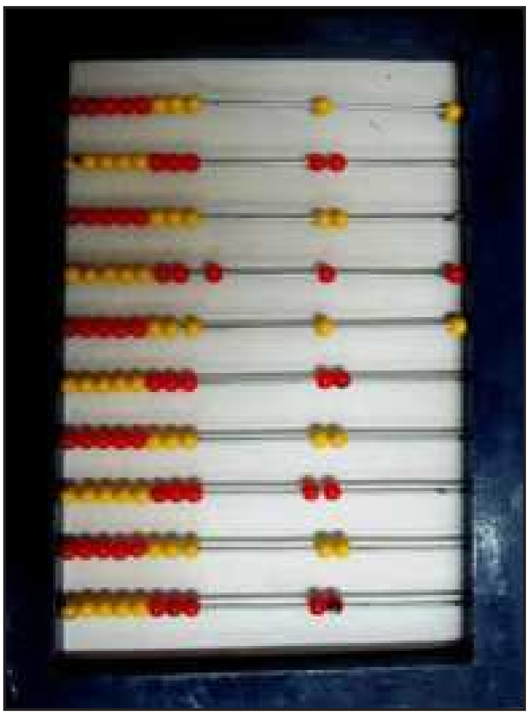
For number 5, 1 bead from the third row and 1 bead from the first.

To represent 7, 1 bead from the third row, 1 from the second and 1 from the first.

To represent any number, the children use the values of single beads in different rows and manipulate them to find their target. Using 1, 2, 4, 8, 16, 32, 64, 128, 256 and 512, each target number is broken, and the rows added mentally. You can imagine how much mental juggling takes place, all the way up to the number 1023.

For practice, ask children to show different numbers on the abacus. Alternately, show numbers on the abacus and have the children guess the value.

25 in code 2

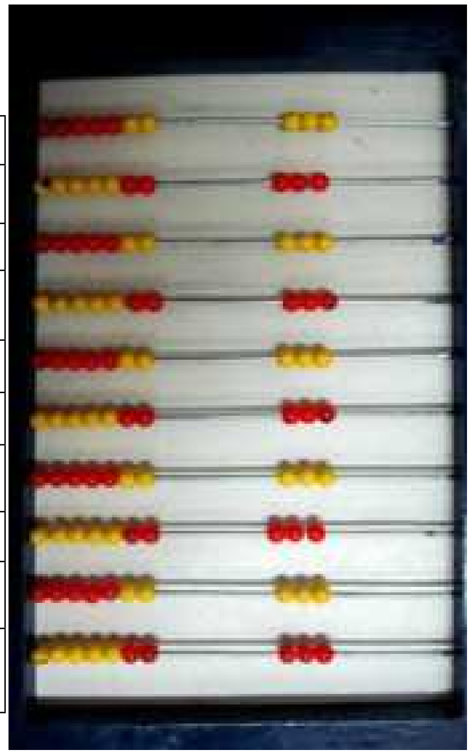


Code 3

Keep three beads in each row. The value of 1 bead in the first row is 1, as in Code 2. The value of 1 bead in the second row, we know, is the total value of all beads in the first row. However, since there are now 3 beads, this value is 3. The value of 1 bead in the third row is same as the total value of all beads in the second row, which is 9.

Therefore the value of a single bead in each of the 10 rows is, respectively, 1, 3, 9, 27, 81, 243, 729, 2187, 6561 and 19683. Take a look at the initial position in code 3.

1
3
9
27
81
243
729
2187
6561
19683



Once the children are familiar with the value of beads in each row, ask them to form different numbers using the abacus. As before, breaking and adding happens mentally, providing good exercise for the mathematical mind.

The number 16 is represented in Code 3. 1 bead in the third row shows the number 9. Remaining 7 is made up of

6 (2 beads in row two) and 1 (1 bead in row one). There is a unique way to represent every number.

16 in Code 3

50 in Code 3

While showing different numbers with code 3, children can use the values of 1 bead or 2 beads in different rows and manipulate them to find their target. Never are all three beads used. In such cases all three beads are pushed back and one bead from the next line is drawn.

Other codes: Like code 2 and code 3, the numbers are also represented with the code of 4, 5, 6, 7, 8 and 9. All these help the children to explore, improve their mental ability to calculate, and get familiar with bigger numbers. Obviously the next (higher) code is introduced only when the children are very confident with the previous code. The rules for handling the abacus for higher codes are similar to the rules of code 2 and code 3. The only change is in the number of beads in each row and the value of each bead.

Addition with Abacus

Take any two numbers in two different abacuses and add one number to the other. While adding, one simply replaces each bead from one abacus and maps it on to the other abacus. In other words, we put back the beads from one abacus and add the same amount of beads in other abacus in the corresponding row. The final result will be the sum.

Example: Suppose you want to add 13 to 15 in the abacus of code 2.

1. Show 13 in one abacus (let us call this abacus A) and 15 in another abacus (called abacus B).

2. Take away the bead from first row of abacus A (showing 13)…

3. …and add it to first row of abacus B (showing 15).

4. Since the first row now becomes 2 beads, it can be shown in the second row with 1 bead.

The second row then becomes 2 beads…

5.…so it can be shown with 1 bead in third row.

6. Now the third row has 2 beads. It can be shown in the fourth row as 1 bead.

7. Since the fourth row now has 2 beads, it can be shown in the fifth row with 1 bead.

8. The bead from third row of abacus A is taken away.

9.… and 1 bead is added to the third row of abacus B.

10. The bead from the fourth row of abacus A is taken away…

11.… and 1 bead added to the fourth row of abacus B.

12. Now all the beads from abacus A have been added to abacus B. In this abacus the beads are in third row, fourth row and fifth row.

The number showing is 28, which is the result of the sum 13+15.

Once children gain confidence with the game using different codes, encourage them to keep the abacus on the floor and calculate just by looking at it (without moving the beads). It will help them to picture the abacus and work mentally. Soon they will be able to calculate bigger numbers mentally without the abacus.

CHAPTER 07

Addition and Subtraction

Addition, Subtraction, Multiplication and Division are the four basic operations in mathematical language. Through these operations we try to imagine the world and interpret it according to our needs. All human beings use these operations in their daily life. In mirambika we endeavour to build a concrete understanding of these operations and their uses in our day-to-day life.

Awareness of the four operations is evident even at a pre-school level, when the child starts playing with different objects such as toys. A child is able to recognise if he has been given one toffee and his sister has been given two. At that time he tells his mother, "Give me one more because Didi has two."

Understanding Addition

We start by playing with different objects collected from the surroundings. A child could be given a set of two leaves and asked to put another set of three leaves together. Then he is asked how many leaves there are altogether. He could be given two pebbles and three pebbles, two flowers and three flowers, two marbles and three marbles, two biscuits and three biscuits and so on. Each time he is asked to put them together and count all. In the next step the number of objects is changed and the child is asked to put these new sets together.

Then he could be asked questions like: How many marbles did you have? How many more marbles did I give you? How many marbles do you now have altogether?

Slowly the understanding of togetherness builds up. The child is then able to express what he is doing and what is happening among these objects.

Slowly the word **add** is introduced in a subtle manner and the child relates the action he is doing with the word **add**. Gradually he includes the word in his vocabulary and is able to use it to describe the action that he is performing.

We do these activities with children of ages 4 to 7. No symbol or sign of addition is used. No written work either. Situations and stories involving addition and subtraction are shared with the children, or created by them. The problems within these stories are what they then work on. Here are a few activities for more clarity.

Addition through Nature walk

Go for a Nature walk. Children love to see beautiful things in Nature like a butterfly, flower, cloud, or feather. They love to collect resources from Nature such as sticks, leaves, feathers, pebbles, twigs etc. We start playing with their collections in this way.

आओ दोस्तों आओ दोस्तों
साथ अपना सामान भी लाओ
मिलजुल कर बैठेंगे
नए खेल हम खेलेंगे।

इहाना, आपके पास छः (six) पत्तें हैं, इशिका के पास नौ (nine) पत्तें हैं। अब बताओ, आप दोनों के पास मिलकर कितने पत्तें हैं ?

आहाना दो (two) फूल लाई है। अभीप्सा तीन (three) फूल लाई है और तारा भी चार (four) फूल लाई है। कौन बता सकता है तीनों दोस्त मिल कर कितने फूल लाए थे?

Some more examples with concrete objects

• Children like to eat laddus. So we ask them, if Sudarshan eats two laddus and Soham eats three, how many laddus do both of them eat in all?
Children take keen interest in such questions, where they are named as participants. Playing out the roles, they put two stones for Sudarshan, three stones for Soham, then add the stones and count them.

• माधव और सुदर्शन एक दिन गिलास रखने के लिए भोजनालय गए। माधव ने पहले सात(seven) गिलास रखे, फिर सुदर्शन ने आठ (eight) गिलास रखे । तो दोनों ने मिलकर कितने गिलास रखे?

Addition without objects

Once children have enough practice with concrete objects, they can begin to visualise without seeing or touching them, i.e. work mentally. We begin with small numbers.

• एक जंगल में चार (four) हाथी थे। एक दिन पानी पीने के लिए गए। रास्ते में उनको आठ (eight) हाथी मिले और उनके साथ दोस्त बन गए। फिर सबने मिलकर नदी में जा कर पानी पीया । बताओ, कुल कितने हाथी पानी पीने के लिए गए थे?

Understanding Subtraction

Subtraction as a process is the reverse of addition. Adding more to a collection of objects makes the collection larger than earlier, but taking away from the collection makes it smaller. To introduce subtraction, we have to think of situations where the children are used to taking away from a bigger collection and are left with a smaller one.

Initially, subtraction is practiced with familiar objects and small stories.

• There were 5 bananas on the plate. You ate three bananas, so how many bananas remain on the plate?

• There were 8 laddus in a packet. Soham took away three laddus. How many laddus are left in the packet?

These are practical situations the children face in daily life, and they relate with them very quickly. Children are also asked these questions while playing with stones and pebbles. Children are allowed to use different objects to find the answer. With plenty of exposure of this kind, they come to understand subtraction. To make children familiar with the language, we keep repeating various terms like 'take away', 'remove', 'reduce' whenever subtraction takes place. Slowly the children associate the terms with the act of subtraction and use them.

• Shireen has seven toffees. She has given two toffees to Riya. How many toffees does Shireen have now? Children may use 7 pebbles to represent the toffees, take two out and see that five are left.

• **जानवरों का मेला** - जंगल में एक बार मेला लगा था। उस मेले में कुछ जानवर घूमने के लिए आये थे । गाय, हाथी, घोड़ा, हिरण, ऊँट, सियार, और बिल्ली । मेले में उन्होंने बहुत सारी चीज़ें देखी और सामान भी ख़रीदे, झूला झूले। सबने अपना मन-पसंद खाना खाया ।

• हाथी, घोड़ा और हिरण ने दो-दो (two-two) आइसक्रीम खाई । अब बताओ, तीनों (three) दोस्तों ने मिलकर कितनी आइसक्रीम खाई?

• सियार और बिल्ली मिलकर सत्ताईस (twenty-seven) लड्डू लाए । हर दोस्त को तीन-तीन (three-three) लड्डू मिले । अब बताओ, सत्ताईस (twenty-seven) लड्डू कितने दोस्तों ने खाए होंगे?

• जब शाम हो गयी, सब इकट्ठे मेले से बाहर आए । सबने हाथ में दो-दो (two-two) गुब्बारे पकड़े थे । उनके पास कुल अट्ठारह (eighteen) गुब्बारे थे। अब बताओ, कितने दोस्तों ने गुब्बारे पकड़े होंगे?

Such stories and questions are aimed at developing listening, thinking, intuition and observation.

Using bundles of sticks for Addition and Subtraction

We use bundles of 10 sticks for addition and subtraction of larger numbers with age group 5 and 6. It helps to break the number into ones, tens and hundreds.

For example, we want to add 24 and 17.

24 sticks can be shown as two bundles of 10 each and 4 separate sticks. Similarly, 17 becomes one bundle of 10 and 7 separate sticks.

Put all the bundles together and all the separate sticks together.

There are 11 separate sticks. Discuss with the children and replace ten sticks with one bundle. Add the bundle to the others. This will leave one stick separate.

The children can clearly see that the final result is 41. After a lot of practice, and some guidence this helps establish the concept of carrying a number during addition.

Let us try another example: 64 + 48

Separate 64 and 48 sticks respectively into bundles of ten and separate sticks.

Add the bundles and individual sticks separately. Now we have ten bundles and 12 separate sticks. The 10 bundles can be rebundled into a big bunch of 100 sticks.

The 12 individual sticks can be replaced by one bundle of 10 and two separate sticks. So, putting it all together, the result is 112.

Note: It should be easy (for the facilitator) to see that the bundles and their placement corresponds to the place values of units, tens and hundreds. This activity therefore builds a concrete foundation for the understanding of the addition of numbers.

Mental mathematics through riddles

Children like to solve riddles. They also have a lot of fun making riddles and playing with friends and Diyas. Here there are some riddles prepared by children and Diyas.

1.

एक पेड़ में हुए थे आम
तीन थे ऊपर
तीन थे नीचे आम
तीन थे बायें और
तीन थे दायें आम
अब बताओ
उस पेड़ में हुए थे कितने आम?

2.

एक टोकरी में दस अमरुद
जितने थे कच्चे, उतने थे पके
अब बताओ,
कितने थे कच्चे और कितने थे पके अमरुद?
आये बच्चे पाँच एक साथ
बाँट लिए आपस में बराबर अमरुद
बताओ,
एक बच्चे को मिलेंगें कितने अमरुद?

3.

पाँच थे बच्चे
दस थे आम
बोलो-बोलो एक बच्चे को
मिलेंगें कितने आम?

4\.

पापा लाये दस सन्तरे
सभी को दिये दो-दो सन्तरे
बोलो-बोलो कितने बच्चों को
पापा ने दिए हैं सन्तरे?

5\.

घने जंगल में रहते थे बारह शेर एक साथ,
लगी उनको बहुत प्यास, चल पड़े सभी एक साथ
मिले उनको रास्ते में, चार और शेर एक साथ,
अभी बताओ मिलकर रहते उस जंगल में
कितने शेर एक साथ?

6\.

पेड़ में जितनी थी लीची
छह बच्चे ले गये दो-दो लीची
अभी बताओ उस पेड़ में
लगी थी कितनी लीची?

7\.

एक तालाब में दस कछुए
रहते थे साथ-साथ
उसमें से दो कछुए
घूमने निकले साथ-साथ
अभी बताओ उस तालाब में
कितने कछुए रह गये एक साथ?

8\.

एक वेल में दस पत्ते
दूसरी वेल में दस पत्ते
तीसरी वेल में भी दस पत्ते
बताओ सब मिलाकर
हो गये कितने पत्ते?

When the 6 and 7 year old children are familiar and confident about doing addition and subtraction with and without objects mentally, we introduce abacus to them to practice with large numbers. Following this, written mathematics and the use of symbols are introduced. After we introduce the symbols for addition and subtraction (at the age of 8+) we play several addition and subtraction card games with children.

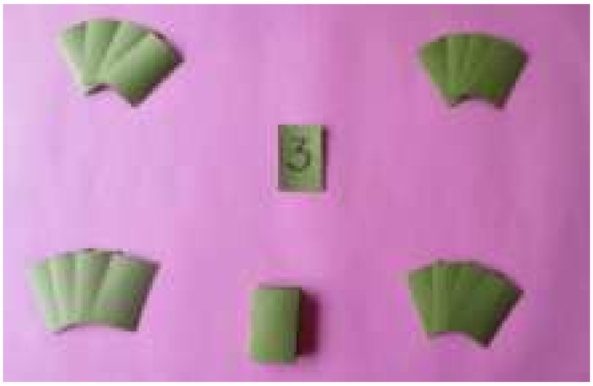
Addition and Subtraction games

Sum of Two	
No. of players	4
Materials	15 cards numbered 1 to 15 (4 such sets, totalling 60 cards)

How to play: The cards are shuffled and four cards distributed to each player. The rest are kept in a pile face-down and one card from the pile is put at the centre facing up. Before the game starts, a target sum (any number from 10 to 20) is decided. The first player checks the cards in hand and sees if there is any card that can combine with the card in the centre to get the target sum.

For example the target is 12. The starting card is 3.

If the player has such a card (for example, 9) then he picks the card and wins the two cards (the 3 and his own 9). He puts the pair aside, then draws



two more cards from the pile, and puts one card at the centre. This is now the card in play for the next player.

If a player has no matching card he draws one card from the pile and discards one card from his hand at the centre. This is now the card in play, and the earlier card is covered and no longer available. (Note that the player may well decide to discard the card that he has just picked up, if he does not find it helpful.)

Then the turn passes to the next player, continuing around the circle till the central pile is exhausted. Then one of the players squares up the pile, turns it over, shuffles it, and places it face down. One card is turned over in the centre and the game continues.

The game finishes when there is no more card in the pile that can be paired to yield the desired sum. The players count the number of pairs that each has collected to determine the winner(s). If desired, another round can be played with a different target sum (again from 10 to 20).

Matching Game	
No. of players	4 or 6
Materials	16 or 24 cards (4 per player) as detailed below

(example cards for a 4-player game)

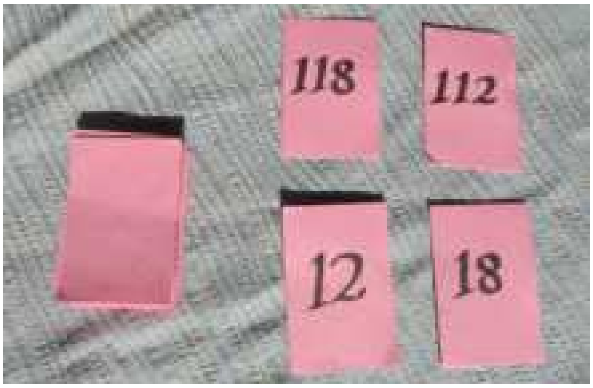
4+5	3+6	2+7	8+1
4+6	3+7	8+2	5+5
5+6	7+4	8+3	9+2
3+9	7+5	8+4	6+6

Note: The number of cards can be prepared according to the number of players. The sums will be written in such a way that 4 cards have the same result. The level of difficulty can be varied according to the ability of the children.

How to play: The aim of the game is to make a set of 4 cards which has the same answer. The cards are shuffled and distributed equally among all the players. Next, each player has to pass one card to the player on his/her right. So the player checks his cards and decides which to keep and which he will pass to try and make a set of 4 cards of equal value. After the decision everyone quickly passes one card to the player on the right (and receives one from the player on the left). Each player then re-checks and decides what to pass next. The passing continues in this way till one player (or more than one) get(s) all four cards having equal value. Then the game restarts.

Dice and Card Game	
No. of players	2-6
Materials	Dice 1: numbered 0, 20, 40, 60, 80, 100 Dice 2: numbered 0, 10, 30, 50, 70, 90 A set of cards numbered in such a way that one can achieve a round number result like 10, 20, 30, 40, 50 and so on till 190 by the sum or difference of two cards.

How to play: More than two players can play this game. Any player can start. Four cards will be placed on the floor facing upward and rest of the cards kept in a face-down pile at the centre.



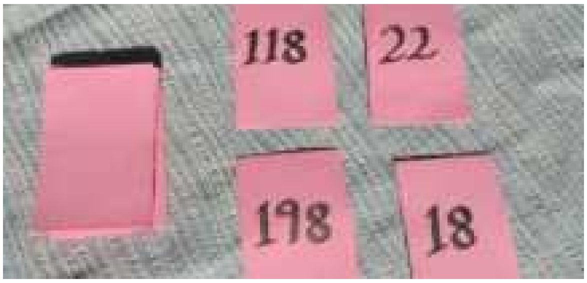
Sample card sets:

2, 12, 22, 32, 42, 52, 62, 72, 82, 92, 8, 18, 28, 38, 48, 58, 68, 78, 88, 98	*12-2=10* *38+2=40*
3, 13, 23, 33, 43, 53, 63, 73, 83, 93, 7, 17, 27, 37, 47, 57, 67, 77, 87, 97	*83-43=40* *97+93=190*
4, 14, 24, 34, 44, 54, 64, 74, 84, 94, 6, 16, 26, 36, 46, 56, 66, 76, 86, 96	*96-66=30* *56+64=120*

The player rolls both dice together, then adds or subtracts the numbers he gets on the dice. (In the example below, he can use 100 (60+40) or 20 (60-40).

Now he must find any two face-up cards that can be added or subtracted to yield the number of his choice (sum or difference of the two dice). In this example, he chooses 112 & 12, and subtracts 12 from 112 to yield 100.

If successful, he picks up both cards. To replace these, two cards are turned over from the face-down pile. In the accompanying picture, we can see that 112 and 12 have been removed, and replaced by 198 and 22.



Play then passes to the next player. If a player cannot find cards to match, play simply passes to the next player. The game ends when the pile is exhausted. If players wish to continue, one of them must square up the pile, turn it over, shuffle it, and place it face down. Play then resumes as before.

In mirambika, the children are introduced to written mathematics at a later age. The games described above help them to become familiar with the numbers through play. Following these experiences, when children proceed on to written mathematics, they understand the concepts better and at the same time, they begin to learn at a faster pace.

In mathematics you are told that the number of elements is finite and that therefore the number of combinations is finite; but that is purely theoretical, for if you come down to practice and all these combinations had to follow each other, even if they went at so great a speed that the change would be almost imperceptible, it is quite obvious that the time needed to make all these combinations would be, apparently at least, infinite; that is to say, the number of combinations would be so immense that no limit could be assigned to it – at least no practical limit; the theory is not interesting for us, but practically it would be like that.

- The Mother

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CHAPTER 08

Multiplication and Division

Multiplication

Basic requisite for multiplication: the child should be able to:

1. Do addition

Addition is the foundation for multiplication. Before learning multiplication, the child must be confident in repeated addition.

2. Make groups of equal size

Make the children familiar with grouping objects into sets of the same size. The children are familiar with different objects like pairs of shoes, pair of hands, three wheels of a tricycle, three blades of a fan, four petals of a flower, four legs of a chair, etc. To make them aware, you could ask them to find or make a list of objects which form such natural groups. They can also be asked to collect seeds, marbles and pebbles and make equal sized sets.

3. Add equal sized groups

Let the children have enough practice of making groups of equal size (using identical objects). Now they can be asked to add some of these equal-sized groups. They can be asked how many toffees are there in three packets of five toffees each. Working with a variety of objects will give them practical experience, which can be reinforced by imaginary tasks.

At this stage the practice of skip counting of 2, 3, 4, 5 etc. can be done thoroughly with objects or bead-strings. Children usually enjoy games related to skip counting.

Bingo	
No. of players	Any number 3 and above
Materials	None

We play this game to practice skip counting. Children sit in a circle and decide first which number is to be 'skipped'. They then count serially in a fixed direction (clockwise or anti-clockwise). Each child simply states the next number, <u>with some exceptions.</u> Each time a multiple of the decided number comes up, the child says bingo instead of the number. The child who makes a mistake loses a point. Play with the 'skipped number' as 2, 3, 4, 5… *(The word 'Bingo' can be replaced by any word by common agreement, like their name)*

While doing skip counting make them aware that what they are counting is in fact

1 times 2, 2 times 2, 3 times 2, 4 times 2 and so on.

1 times 3, 2 times 3, 3 times 3 and so on.

Lots of skip-counting practice with different objects, games and stories makes the children proficient in tables. There is no need to memorise tables like a parrot.

Understanding of multiplication

Example: We could begin with a story. On his birthday, Soham decided to distribute toffees to his friends in the group. That day 15 children were present, including Soham. He thought he would give 4 toffees to each of his friends and also to himself. He asked his mother to buy toffees from the store. His mother wanted to know how many toffees to buy. Now Soham started thinking.

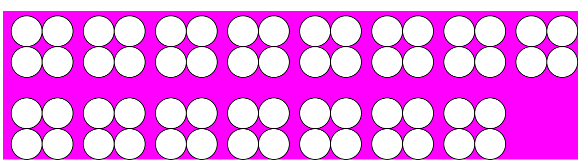
Ask the children if anyone can help Soham to solve his problem. Give some beads to all the children separately and ask them to help Soham if they want to eat 4 toffees each.

This is what happened in one instance:

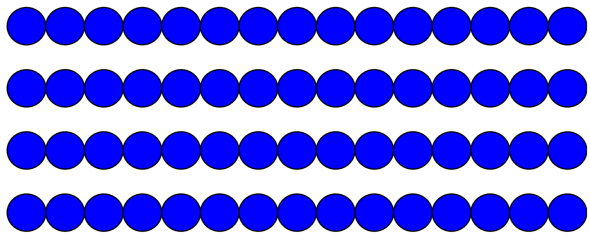
All the children got busy with counting. After a while, many arrived at answers. Some arranged their beads in 4 groups with 15 beads in each group and some children arranged it in 15 groups with 4 beads in each. They added all the beads and their answer was 60.

Here the teacher can ask the children how they got 60. Why did they add?

One child said that each child would get 4 toffees and there were 15 children present in the group. So there would be 15 groups, each having 4 toffees. To find the total number of toffees we would need to add all 15 groups of 4.



Another child said, "If I give 1 toffee to each, I need 15 toffees. If I give 2 toffees each, then I need 15+15 and that is 30 toffees. If I give 3 toffees each then I need 30+15 and that is 45 toffees. If I give 4 toffees each then I need 45+15 that is 60 toffees. That means if I give 4 toffees to 15 children then I need 4 times of 15 toffees that is 15+15+15+15. So altogether I need 60 toffees."



You can see how clear the children are in their thought process when dealing with practical situations. You need only to guide and create a situation for their learning. Now discuss with the children the relationship between multiplication and addition. Use various examples, involving small numbers. The children should perceive that multiplication is nothing but repeated addition of the same number.

When we think about multiplication, many parameters related to multiplication come to mind:

1. **Equal grouping**: In this type of situation, we want to find how many objects there are in several groups of equal size. Example: There are 3 plates with flowers – 5 roses, 5 jasmines and 5 marigolds respectively. How many flowers are there in all the three plates?

2. Array: To find the total number of objects when the objects are arranged in a rectangular pattern of rows and columns. Example: There are 6 rows of rose plants in a flower garden. Each row contains 8 plants. Find how many rose plants there are altogether in that garden.

3. Scale (multiplying factor): When the cost or value of an item is given and we have to find what it will be if it increases by a certain number of times. Example: the cost of one pencil is 5 rupees. A fountain pen is expensive and costs 3 times the cost of a pencil. What will be the cost of the fountain pen? Exercises like this are helpful for the children in bringing the shift from mental mathematics to written calculation.

Since we start written mathematics at the age of 8, before that it is enough to give them sufficient practice in mental multiplication. When the time comes for written mathematics, simple problems such as the following are best to start with:

You ate 2 toffees, your friend ate 2 toffees. How many toffees did you eat altogether?

Child: Four toffees.

Diya : How did you know?

Child: I added 2 and 2.

Diya: That means you added 2 two times. So how we can say that?

Child: 2 times 2 toffees.

Diya: Yes, very good!

How can we say it?
Child: 2+2 =4
Diya: Yes, the other way to say 2 times 2 is 2x2
Similarly 3 times 2 = 2+2+2=3x2
Now can you tell how we would say 4 times 2 toffees?
Child: 2+2+2+2= 4x2
Diya: Yes, so the symbol 'x' is used for a number of times.

Exercise: How can you express the following?

3 times 2
4 times 5
6 times 3

The children should be clear that multiplication is the repeated addition of a group of numbers of the same size.

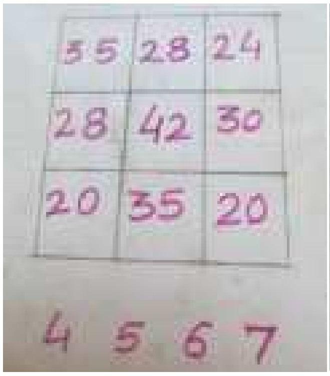
Once they are clear about this, they can do multiplication with bigger numbers. At this stage, several games can be introduced to help them with the multiplication tables.

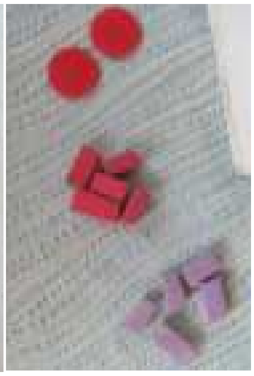
Games for practicing tables

Connect Three	
No. of players	2
Materials	Cardboard/slate 5 pointers having two different colours (two sets) 2 markers

Preparation: Draw a 3x3 grid on the cardboard or on the slate. Choose any four numbers between 2 and 9 and write them below the grid. Write all possible products of the numbers chosen inside the grid. Any three numbers will be repeated in the grid.

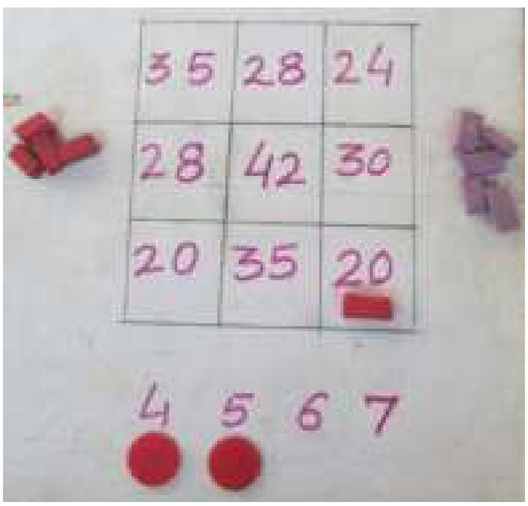
How to play: This game is like Tic-Tac-Toe (also called noughts and crosses).





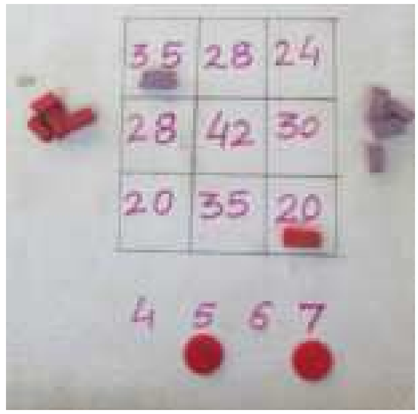
Each player has a set of identically coloured pointers.

One player starts the game by placing markers on any two of the numbers written below. In our example, 4 and 5.



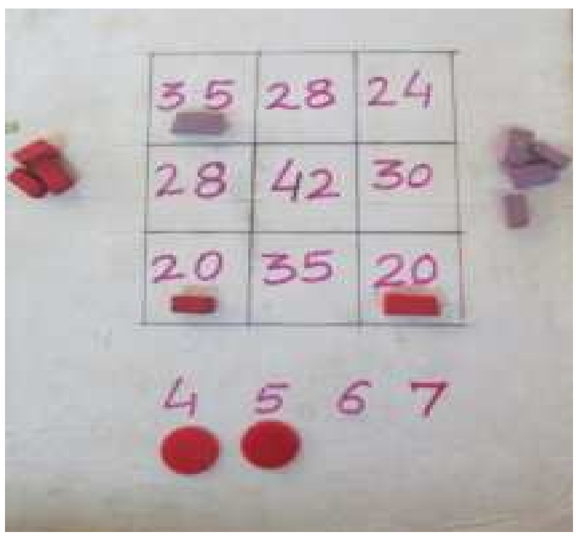
He finds the product of the two numbers in the grid and puts a pointer on it. In our example, 20, as shown.

Then the second player moves any one of the markers to another of the numbers written below. (In our example, he moves a marker from the 4 to the 7). He then finds the product of the two numbers marked and puts his pointer on the matching number in the grid.



The first player then takes his turn. (In our example, he moves the marker back to 4, trying to make a horizontal line.

The winner is the player who succeeds in placing three of his pointers in a straight line (Vertical/ Horizontal/ Diagonal).



Dice Game	
No. of players	2-6
Materials	Counters Dice (two) Cards

Preparation: Make two dice, each with any 6 numbers from 2 to 9. (The dice need not be identical. In fact, choosing a different set of 6 numbers for the second dice will give the child more numbers to play with.) Make a set of cards out of chart paper. For the numbers on the dice, write each possible product of the numbers on a card. Make sets of six counters for each player out of colour chart or foam (of any colour).

How to play: All cards are placed on the floor, face up. Each player gets 6 counters. To start, one player rolls both. (In our example, he obtains 4 and 5)

He multiplies the two numbers rolled, and places a counter on the card bearing the product.

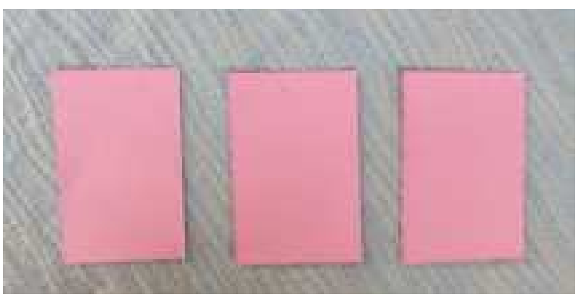
Play passes to the next player.

If a player places his counter on a card that is previously occupied, the player has to pick up the previous player's counter and adds it to his pile. The player who exhausts all his counters first is the winner.

Guessing Game	
No. of players	2 or more
Materials	Set of cards numbered consecutively from 1 to 9

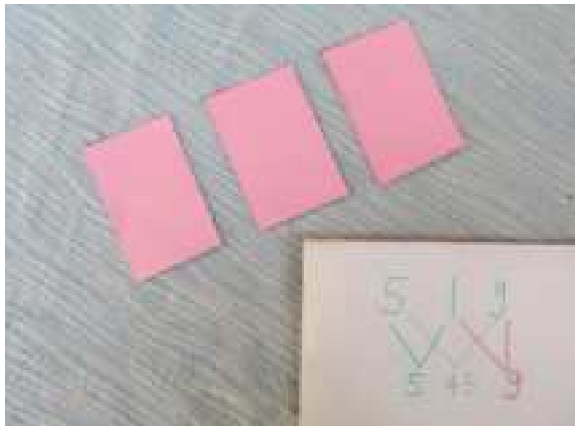
How to play: One player (may be the facilitator) is the leader. She shuffles the cards properly and draws any three. She shows the other players these three cards, keeping the other cards aside.

She then mixes the three cards in front of the players and places them face down in a line calling them 1^{st}, 2^{nd} and 3^{rd}.



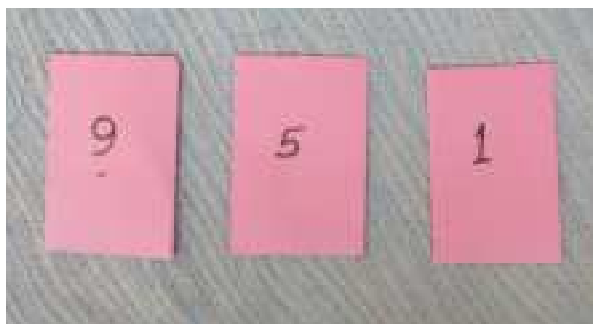
The other players guess the numbers on the cards and find the product of all pairs. They write their answers on a piece of paper in the format shown:

For example, one player guesses: 1^{st}= 5, 2^{nd}= 1 and 3^{rd}= 9. His product will be 1^{st} x 2^{nd}=5, 1^{st} x 3^{rd} =45, 2^{nd}x 3^{rd} = 9



At the end, the leader exposes all three cards and the players check their results.

In our example, if the order of the cards revealed is 9, 5, 1, then the products would be 45, 5, and 9.



Anyone who guessed correctly and identified the cards will get one point. Then a new leader starts again with all 9 cards.

Played often, such games help children to understand the concept of multiplication and become confident with tables. They can then proceed to the multiplication of bigger numbers.

Division

A lot of children find division difficult. They are not familiar with this operation in their day-to-day life. We must try to introduce division by first widening their experience. It is important that the child should understand what division means and its uses in daily life. Like other operations, division should be introduced to the children through activities, stories and games with concrete objects.

Activity 1: Give 20 beads/pebbles to each child and ask them to divide the beads into groups of 4 and count the number of groups.

Activity 2: There are 18 children in Orange group. At sports time in the morning, didi wants to make 3 teams of equal size. How many children will be in each team? (Ask the children to divide 18 children into 3 teams)

Activity 3: Manav had 12 toffees. He shared the toffees equally among 4 friends. How many toffees did each one get? (Give 12 objects and ask them to divide into 4 groups)

In this way, the children can play with objects like pebbles, beads, bottle caps, while solving problems in real or imaginary stories. To help with understanding the concept, use multiple terms like share, divide by, divide into, how many groups, divide equally and split into equal parts.

I. Division is repeated subtraction of groups of the same size

It is only when children are able to divide the given objects through different stories do we begin the process of understanding the operation. We start with a game called Guessing game.

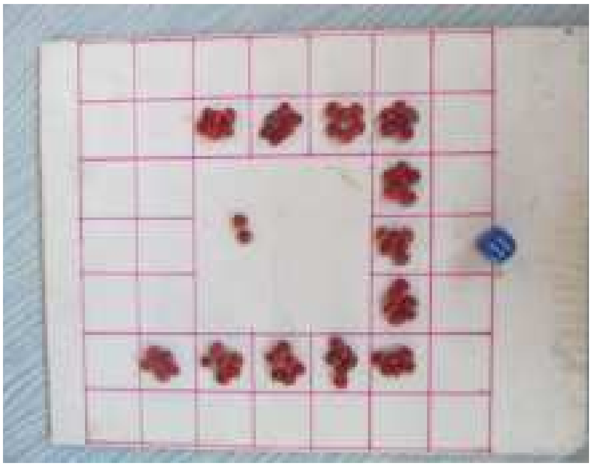
Guessing Game	
No. of players	3 or more
Materials	About 100 counters (beads, seeds, pebbles, buttons) Container for counters Regular game dice numbered 1 to 6 Game board showing rows and columns

Aim: To develop the faculty of estimation and to understand the meaning of division.

How to play: In each round, the leader takes a handful of counters and places them at the centre of the game board.

He then rolls the dice and announces the number thrown on the dice. In this example, 6.

All other players now guess the number of groups that will be formed when the counters are divided into groups having as many counters as this number. They will also guess the number of counters that will remain. After all the players declare their guess, the counters will be separated into different groups. Each group of counters will be placed in one small square on the game board. In our example, the counters are divided into groups of six and each group is placed within one square on the board. There are 12 groups of 6 counters and 2 that are remaining, which makes 74 counters in all.



When there is no possibility of making more groups, the leader will declare the result. The player that guessed correctly will get one point. The game will continue like this. Every time the leader will change the amount of counters in the centre pile.

Each time an individual child is encouraged to separate the counters and keep it in a square. While doing this the children are subtracting the same number of counters from the pile of counters repeatedly. At the end they are counting the number of groups they were able to form. After playing for 30 to 45 minutes, ask the children, “what were you doing to check your gussess?” The children give interesting answers. The more they play this game, the more their estimation power becomes accurate. Eventually, halfway through the game they calculate mentally and tell the answer.

Note: For division by larger numbers,

make a dice with numbers higher than 6 (say, 7, 8, 9, 10, 12, 15)

The children give interesting answers. The more they play this game, the more their estimation power becomes accurate. Eventually, halfway through the game they calculate mentally and tell the answer.

II. Division is the Reverse of Multiplication

When the children are ready for written mathematics, the mathematical symbols and concepts are introduced through different activities and games.

To understand this concept, we perform an activity.

Diya: When 10 pencils are shared equally among 5 children how many pencils will each one get?

Child: 2.

After getting the answer Diya writes 10÷5=2

Diya : If 2 pencils are given to each child, how many pencils will be required for 5 children?

Child: 10

Diya will write 5x2=10

3 or 4 such related questions can be asked, all written on the board.

For example,

10÷5=2	5x2=10
8÷4=2	4x2=8
12÷3=4	3x4=12
15÷5=3	5x3=15

Now ask the children to find out the relationship. During discussion the children come out with many answers like:

- In both the problems the numbers are same but position is different.
- The first number in the first problem is same as the last number in the second problem.
- The answer of the second problem is the question of the first problem.

Finally, the discussion comes around to the observation that multiplication and division are the reverse of each other. Gradually, with such activity and a lot of practice with objects, when children are asked to divide 8 by 2, they try to reason in the reverse order, 'How many times does 2 go into 8?' Some recite the table of 2 till they get to 8. Some repeatedly subtract 2 from 8.

Remember, all of these are okay. It is common for the child to use an approach different from the facilitator's own. It is important to respect the child's intuitive approach, only ensuring that the underlying concept is being understood.

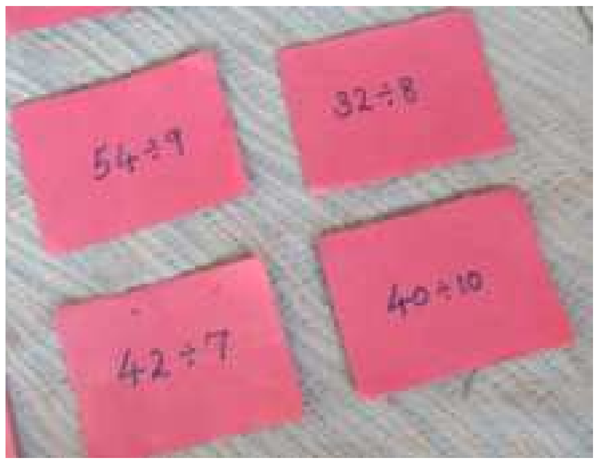
After the children understand the concept and process of division, they need practice to speed up their calculation. To practice division we introduce a few games.

Matching Game	
No. of players	4 or 6
Materials	A set of cards (4 times the number of players) such that four sums have the same result

Sample set of 16 cards for 4 players. The level of the game can be graded according to the children's ability.

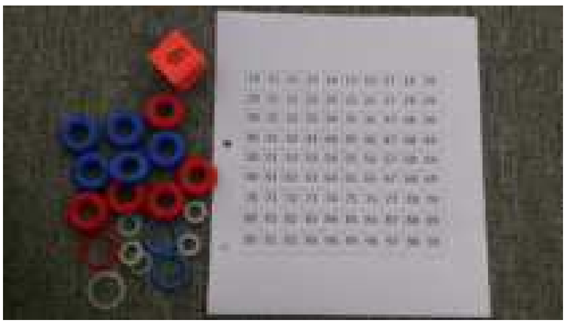
10÷2	15÷3	20÷4	35÷7
15÷5	18÷6	24÷8	27÷9
16÷4	24÷6	32÷8	36÷9
14÷2	28÷4	35÷5	42÷7

How to play: The aim of the game is to make a set of 4 cards which have the same answer. All the cards are shuffled and distributed among the players. Each player checks his cards and decides which card he will pass on to try and make a set of 4 cards of equal value. Everyone passes one card to the player on their right and receives one card from the player on their left. After this, each player re-checks and decides what he will pass next. Another round of passing follows. This continues till any one player gets all the four cards having equal value. The winner gets one point. Then the cards are re-shuffled for another round.



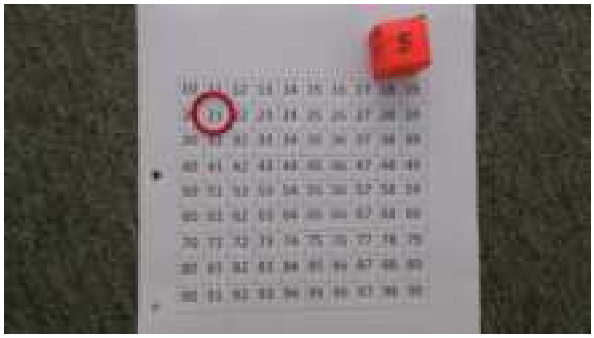
In this example, the player has four cards, two each with results 4 and 6. He has to decide whether he will discard a 4 or a 6, then hope to receive passed cards to support his chosen result.

Remainder Game	
No. of players	4
Materials	Board with 9x10 grid having the numbers 10 to 99 sequentially (as shown) Dice with any six numbers 2 to 9 Small hoops made of any stiff material, such as wire, or readymade plastic rings



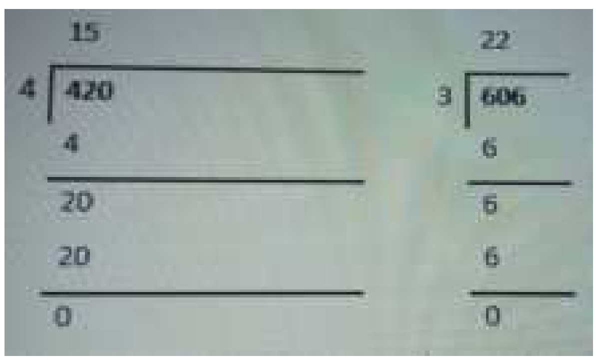
How to play: The first child chooses a number from the board and encircles it with a hoop. Then she rolls the dice and divides the encircled number by the number on the dice. The remainder obtained from the division is the score for the player. Like this, all the players will get a chance. One cannot choose a number that is already played. Play will continue till all the numbers have been played. At the end, all the players' scores are added.

In our example the player has encircled 21, then thrown a 5 on the dice. Dividing 21 by 5 leaves a remainder of 1, which is the player's score for this turn.



III. Algorithm for Division

Once the children understand the concept and process of division, and get enough practice, they will be able to divide small numbers easily. But to divide large numbers they need an algorithm, as well as comfort with writing. As a preparatory exercise, introduce written work using small numbers, such as dividing a one-digit number by another, then a two-digit number by a one-digit number. When the time is right, introduce the long division method and larger numbers. However, watch out for errors such as these:



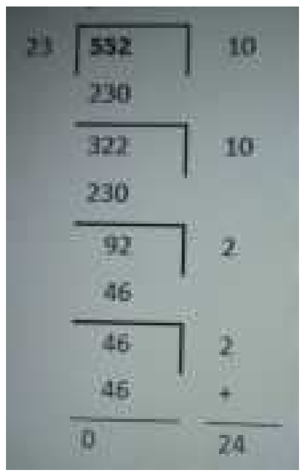
Thus the answer they get is 15 and 22 rather than 105 and 202, respectively. The child here does not see 420 as a whole but has divided it into parts and therefore commits these errors. This is a result of mechanical learning of the algorithm, without grasping the underlying concepts.

We could do a small story to get them to divide large numbers without following this long division method. This will help them get a better sense of the numbers.

There are 552 people in a dangerous place. To escape they need to shift to another place as quickly as possible. The leader of the kingdom plans to rescue them and sends an airplane. However, the capacity of the airplane is 23 plus one pilot. How many times does the airplane have to go to and fro to shift all the people to safety?

While doing this question in the classroom, children did not know the multiplication table of 23 but still tried to find the answer. Some of them tried to subtract 23 repeatedly from 552 and checked how many times they could do it. Some children tried to see how many people could be shifted if the plane went 10 times. They arrived at 230. They saw that if the plane went another ten times they could shift another 230 people. After 20 times the remaining people will be 92. If the plane will go 2 more times it will shift 46 people. So the remaining 46 people can be shifted in 2 more goes. So the total number of rounds for the plane is 24.

In this method the child is seeing the whole number of people at a time and subtracting the number of people repeatedly from the total. Here he does not need to remember the table of the divisor as we do in regular long division method. He only needs to know the 10, 100, 1000 times of any number and double and half of the number.



The children are guided step-by-step in this way to understand the concepts and solve word problems related to the four operations. During Math Melas, children create their own mathematical puzzles and board games for children from other groups.

(Concerning a choice of textbooks for a mathematics class)

The teacher or teachers should use the book to prepare lessons that are adapted to the knowledge, the capacity and the needs of the students. That is to say that the teachers should learn what is in the book and transcribe it and explain it to the students, bit by bit, a little at a time, with plenty of explanations, comments and practical examples so as to make the subject accessible and attractive, that is, a living application instead of dead, dry theory.

- The Mother

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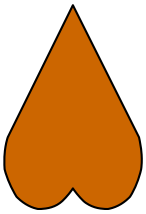
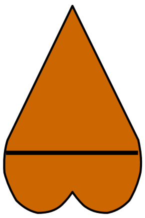
CHAPTER 09

Fractions

Is it really half?



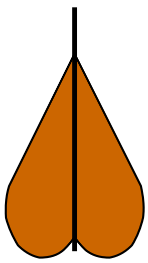
To explain this concept of half, we as usual start with a story. Once father brought a pack of biscuits of different flavours and different shapes. He asked Samara to share each biscuit with her 6-year-old younger brother, Shankar. Samara took one biscuit which was heart-shaped like this.



Samara cut into two pieces like this and gave one piece to Shankar. But Shankar cried and complained to his father that didi had not given him half.

Diya: Do you think that it is really half?

Children: No

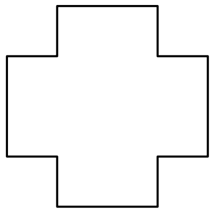


Diya: Come and show how it can be halved.

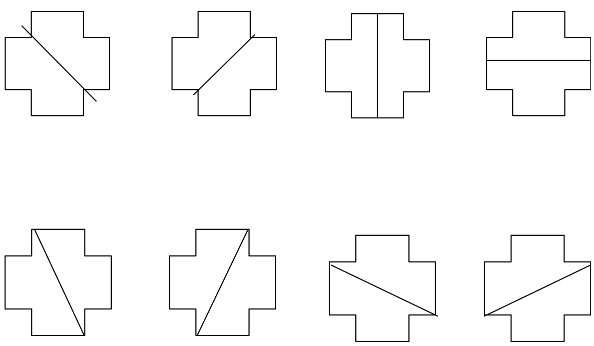
Some of the children are able to cut it into two halves.

Diya now shows different shapes and asks the children to break these into halves. You will see children come up with a variety of ideas on how to make halves. Discuss every idea with them in the class. The discussion enables children to learn from each other and to look at the activity in many more ways. If there are mistakes, do not point out the error. Mistakes often enable more learning. It also helps the Diya to understand where the children are getting stuck and decide how to go further. Simplify question further till the children are able to work it out themselves.

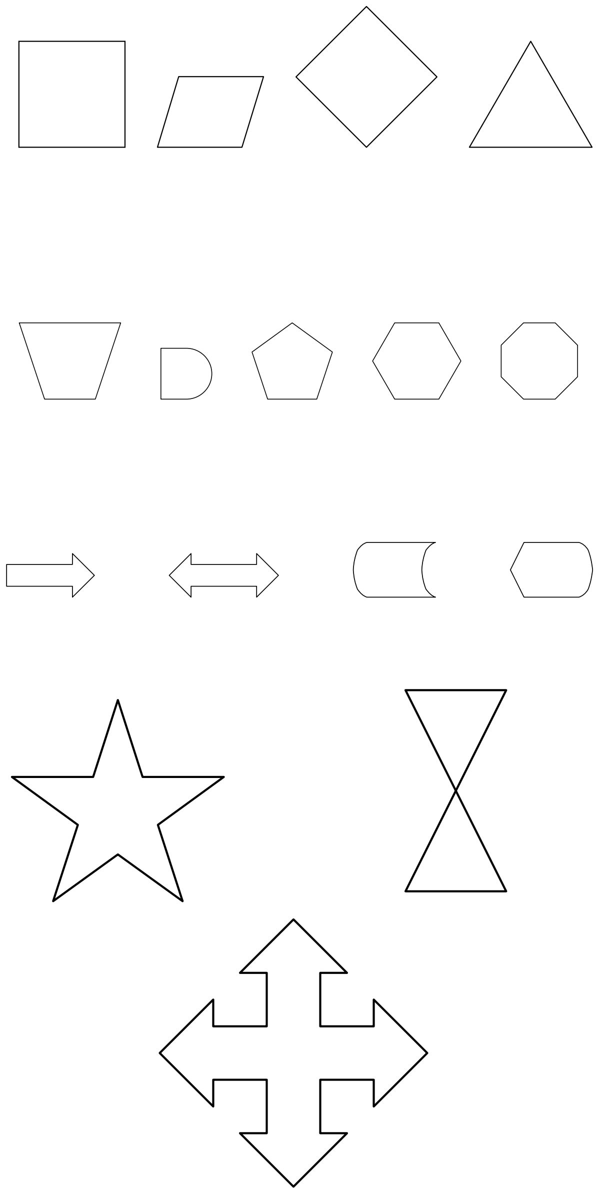
Example: Draw and cut out multiple copies of this shape (around 10 for each child). This is to be divided into two halves in different ways, using a single cut. (8 possible ways are shown below).



All responses should be accepted. Ask the children to find ways of comparing the two and confirming that they are of the same size. This will help them to eliminate invalid answers.



Activity: Give a sheet with drawings of different shapes. Ask the children to divide these into halves using a ruler and pencil, colour one half, then write 'half' under the picture.



Activity: Hand out a piece of paper and ask the children to divide it into two equal halves. See in how many ways they can do this.

Introducing Denominator

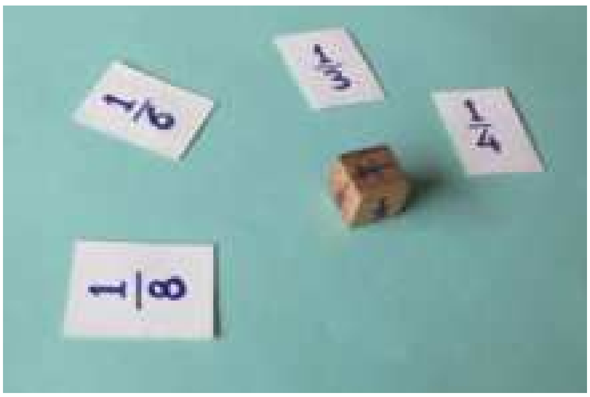
Viren likes cake very much. All the children of the group decide to invite Viren for a surprise cake party on his birthday. There are sixteen children in the group other than Viren. So they arrange four tables in an interesting way. At the first table, there are 8 chairs, at the second 6 chairs, 4 chairs at the third, and only 2 chairs at the fourth table. The children are waiting for Viren and they have left one chair free for him at each table. Each table has a beautifully decorated, yummy cake. Now Viren arrives a little late. The children are asked to guess where Viren will sit and why?

The more the number of people at a table, the smaller is their share of the cake. The children arrive at this conclusion. Now we introduce the symbol of fractions 1/2, 1/4, 1/6 using the story.

Activity: The children are given different shapes like circle, square and rectangle made of chart paper and they are asked to show 1/2, 1/3, 1/4, 1/5, 1/6 and 1/8 by drawing lines, colouring one part as asked, and pasting it in their notebook. After the activity they can see each other's work to observe the different ways of expressing the same fraction.

Denominator Game	
No. of players	4 (can also play with 3)
Materials	8 cardboard circles, cut into equal parts as shown Dice: (1/3, 1/4, 1/6, 1/6, 1/8, 1/8 4 cards: (1/3, 1/4, 1/6, 1/8)

Cut out 8 cardboard circles of the same radius. Then cut one circle into quarters, one into thirds, one into sixths and one into eighths, as shown below in the picture. Do the same for the other four circles.



How to play: Put all the fraction pieces of 1/3, 1/4, 1/6 and 1/8 at the centre. Shuffle and distribute one card to each player.

The player who starts the game rolls the dice.

If the dice shows the number she has on her card, then she picks up one piece of the same fraction from the pile of fractions at the centre. Each player gets a chance to roll and collect her fraction pieces. If the number on the dice does not match her card, she misses the chance to pick up a fraction piece and must wait for her next turn.

Each time a player picks up a piece, she arranges it to try and make a full circle.

With practice, children will be able to distinguish between 1/6th and 1/7th or 1/8th. A smaller piece like the last one used will not complete the circle.

The outcomes of this game:

• The children will become familiar with the fraction notation.

• They can compare two fractions like 1/4 and 1/6.

• They find each fraction needs as many cards as its denominator to be a whole circle.

• They will realise that if we pick a card of 1/8 instead of 1/6 it will not fit into the circle.

They can distinguish between like and unlike problems.

All these things will emerge from discussions with the children after playing the game often enough. While discussing, we should emphasize reading the fraction 1/3 as one-third , 1/4 as one-fourth, 1/5 as one-fifth, and so on.

We may need to point out this important difference:

'a fifth' — one part out of five parts of a thing or a whole.

'fifth' – Item number 5 in a sequence of 5 or more.

Activity: After playing the game, you can give an exercise – to order unit fractions in both ascending and descending order with fraction pieces. The children can label each piece by writing below it.

Introducing Numerator

Sample discussion with children:

Diya: There are 3 children sitting at one table and 6 at another. On each table is kept a pizza to be shared by the children at that table. Tell me, is it fair to all the children?

Child: Not at all.

Diya: What can be done to make it fair, if the children are not allowed to change their places?

Child: Give one more pizza to the table of 6 children.

Diya: The children in the second table will get 2 pieces of 1/6.

Child: Yes

Diya: Then how will it become fair?

One child comes forward and shows by joining two pieces of 1/6 and putting on 1/3.

Child: See how it is fair.

Now the discussion extends to focus on numerator. The children are told that 2 times of 1/6 is written as 2/6. Like this 3 times of 1/4 is written as 3/4

Numerator Game	
No. of players	4 (can also play with 3)
Materials	Fraction pieces (as used in de-nominator game) Dice: 1/3, 1/4, 1/6, 1/6, 1/8, 1/8 Set of cards: 2/3, 3/3, 4/3, 2/4, 3/4, 4/4, 5/4, 2/6, 3/6, 4/6, 5/6, 6/6, 7/6, 3/8, 4/8, 5/8, 6/8, 7/8, 8/8, 9/8

How to play: The fraction pieces are kept in the centre. So are the cards, face-down in a pile. Each child takes one card and places it face up in front of him/her.

The first child rolls the dice. If the dice shows the denominator given on his card then he picks up one piece, to start building the number on the card.

If the denominator in the dice does not match with that of the fraction on his card, he misses the chance of picking a piece, and the turn passes to the next player.

When any player completes the fraction he wins the card.

He then returns all his fraction pieces, and draws another card from the pile. Play proceeds till the pile of cards is finished. The winner is the one who has won the maximum number of cards.

The outcome of this game:

Children develop the understanding that 5/6 means 5 times of the fraction 1/6.

Activity: Different numbers like 2/4, 3/4, 2/6, etc., are shown to the children (either using card games or writing on the blackboard). Children try to join different fraction pieces to get to the target number.

Activity: The children draw circles in their notebooks, then mark and colour different fractions.

Activity: The children make half, quarter by using fraction pieces, i.e. 1/2 =1/4 + 1/4, 1/2= 1/6 + 1/6 + 1/6 and so on.

Activity: To make one-half, one-third, one-fourth by using different kind of fractions pieces.

i.e. 1/2 = 1/4 + 1/8 + 1/8

1/3 = 1/6 + 1/12 + 1/12 and so on.

Activity: Children draw circles and squares, then mark and colour different fractions.

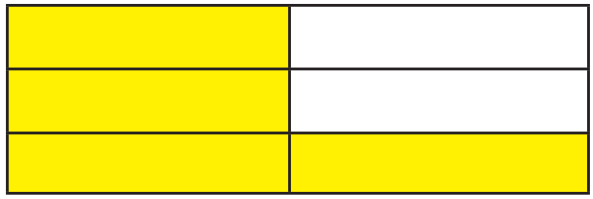
Fraction of a set of objects

After playing sufficiently with fraction kits, the children will be familiar with different fractions and what they mean when a whole object (like a circle or a cake or an orange) is to be divided into equal parts. Now we proceed to the next step.

We start with an example. We give each child a set of 12 beads, seeds, stones or bottle caps. We ask them to divide it into 2, 3, 4 or 6 equal parts. Now we ask them to show 1/2, 1/3, 1/4 and 1/6. We ask them to count 1/2 of 12, 1/3 of 12, 1/4 of 12 and so on. We give them several different sets of objects to create a context for practice.

Activity: Children make grids of 2x3, 3x4, 4x5, and so on, then colour different fractions.

For example



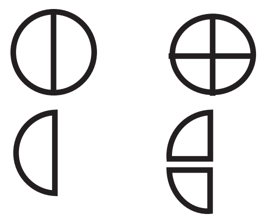
4/6

Activity: We ask the children verbally – what is 1/3 of 27, 3/4 of 16 and so on.

Activity: Written work: To find out various fractions of different sets of objects.

Equivalent fractions

Two fractions are said to be equivalent if they have the same value. For instance, ½ and 2/4 are equivalent because two quarters (2x1/4) are same as one half (1/4).



½ (2 x 1/4) 2/4

The child should experience that one fraction can be shown in various ways.

Activity: The children are asked to make a given fraction, say 1/2, by using fraction pieces of only one type. For this example, children can show different ways – 2 pieces of 1/4 or 3 pieces of 1/6 or 4 pieces of 1/8.

If the children are familiar with this concept, ask them to write ½ in different ways that are equivalent. For example, 1/2 = 2/4 = 3/6 = 4/8… (Once the children get the idea, they will enjoy writing huge numbers. Be patient and always encourage them, however large their numbers get!)

Repeat both these activities for other fractions, such as 1/3, 1/4, 1/5, etc.

Follow up with a discussion with the children, to clarify their learning and make them more confident about representing a fraction in many ways. The discussion concludes with the question "What is an equivalent fraction and how can we find an equivalent fraction of any fraction?"

Paper Folding Activity

To show 1/3 = 2/6

Step 1: Take a piece of paper and fold it into three equal parts and shade one part, that it 1/3.

1/3
1/3
1/3

Step 2: Take another piece of paper of the same size and fold it into 6 equal parts. Shade two parts (2/6), which is the same as 1/3rd of the same piece of paper. This means 1/3 = 2/6

1/6	1/6
1/6	1/6
1/6	1/6

Like and Unlike Fractions

Like fractions are those which are made out of a number of same pieces of fraction. For example: 2/4 and 3/4 are like because 2/4 is made of 2 pieces of ¼ and ¾ is made of 3 pieces of ¼. Another way of looking at it is that the denominators of like fractions are the same. On the other hand, examples of an unlike fractions are 2/4 and 3/5. Here the denominators are not the same or the number of divisions or shares of the fraction are not the same.

Like Fractions

As usual we begin with a story. Here is how a conversation could go:

Diya: In the morning Rini ate a quarter of a cake and her brother ate another quarter of the cake. Tell me how much cake did both of them together eat?

Now the children take one piece of 1/4 for Rini and another piece of 1/4 for her brother and put them together.

Diya: What did you get after putting the two pieces together?

One child: Two pieces of 1/4.

Diya: How would you say that in fractions?

Child: 2/4

Diya: Very good. Is there any other way to say 2/4?

Another child: Yes, 1/2.

Diya: Excellent! Now who can show how to write it mathematically?

Child: 1/4 +1/4 =2/4 or 1/2

Diya: You can write, 1/4+1/4 =2/4=1/2

Like this, the Diya could keep asking many questions related to daily life, and the children add/subtract with fraction pieces.

Unlike fractions

Kuhu ate 1/3 of an orange and her mother ate 1/6 at breakfast. Do you think there is any orange left for her father? If yes, how much?

Now the children are given fraction kits. They put one piece of 1/3 and one piece of 1/6 together and say that it is half (1/2). But earlier they had played lots of games with fraction pieces and they already know that 1/3 is double of 1/6. So while adding with the help of Diyas, they can put two pieces of 1/6 in the place of 1/3 and add it.

Like 1/3 + 1/6

2/6 +1/6 = 3/6 = 1/2

After seeing the result of the addition the children can say easily that there is half an orange left for Kuhu's father.

After a lot of practice with fraction pieces, the children recognise that one fraction has to be converted into its equivalent fraction to make them both 'like fractions'. Gradually the children are encouraged to add two unlike fractions by making them like fractions.

For example, add 1/4 and 3/8

To add these two fractions we make them like fractions by making their denominators the same. Now it is easy to find that the equivalent fraction of 1/4 is 2/8. Having done this, 2/8 and 3/8 are now like fractions and we can add them without any difficulty. One can proceed with subtraction in the same way.

There are also sums like 3/4 + 2/5 in which one needs to change both the denominators to make them like fractions. In that case, we find the LCM (Least Common Multiple) of the two denominators. The LCM becomes the denominator of the like fractions. (When the children are ready to understand LCM and HCF (Highest Common Factor), then we take that up in a separate topic and introduce and practice such additions.

Example, 3/4 + 2/5

Multiplication of Fractions

Multiplication of a fraction by a whole number

The children are well acquainted with the multiplication of whole numbers with unit fractions while playing the numerator game. They already know that 5 times 1/8 is 5/8. Now we are going to discuss composite fractions.

We start with this story: On her birthday, Risaa invited three of her friends to her home. Her mother made a cake. After cutting the cake, Risaa distributed 2/12 of cake to each of her friends. Can you tell how much cake the three friends ate altogether?

The children engage with fraction kits to work this out. We see some children joining two-pieces of 1/12 and keeping these in three separate place, while some are putting it all together.

Diya: Anyone get it?

Child 1: Bhaiya, they ate 6 pieces of 1/12 of a cake.

Child 2: Bhaiya, they ate 3 times 2/12 of a cake.

Diya: Very good, tell me what is 3 times of 2/12?

Child: 3 times 2/12 is 6 times 1/12

Diya: Okay, what is 6 times 1/12?

Child: 6/12

So the children can see the objects and find the answer themselves. They only need guidance through the right questions. Like this, many stories are used to understand multiplication.

Written work with symbols is done alongside.

Like 3 times 2/12 is equal to 6/12

Using mathematical notation, this is written as 3 x 2/12 = 6/12

At this stage, it is more important for the children to internalise the process of multiplication. Simplification can come later.

Mxed Fraction

The children easily understand mixed fraction when it is given to them with relevant real-life examples, such as:

My pencil is 1 ½ times longer than yours.

My father's weight is 2 ½ times of your father's.

We need to help them to use this in their calculation. We could start like this:

Vishal's weight is 24 kg. His father's weight is 2 ½ times Vishal's. Find the father's weight. Here, the children calculate 2 times of 24 is 48 and ½ of 24 is 12. So the required weight is

48+12= 60 kg.

Multiplication of a fraction by a fraction

Generally, children get confused when they are asked to find 1/2 of 1/2 or 1/4 of 1/2 which involves multiplication of fractions.

We can use paper folding to help the child understand multiplication of fractions. For example, finding out what is 1/3 of 1/2 using a sheet of paper.

Take a sheet of paper and fold it into half as shown in figure 1 below. Unfold it. You have two halves of the paper. Now fold it again in thirds as shown in figure 2. Unfold it. You will find that the two sets of folds have resulted in the paper being divided into sixths as shown in figure 3. This can also be written as 1/3 X 1/2 = 1/6

Figure 1

1/2	

Figure 2

1/3

Figure 3

1/6	

Note: *What do we mean when we say ½ of something? What does this 'of' mean? In language, when we have two of something, say, apples, that simply means two apples. Two of an apple means two apples. In other words, 2 multiplied by 1 apple, or 2 x 1 apple. It is the same with fractions. Imagine 6 apples in a box. What will half of this box be? That's simple enough (if we ignore the actual box) – 3 apples. That is the same as ½ X 6. Similarly, if we take half an apple, what will half of that be? We get a quarter apple, which is the same as ½ X ½. So, whether it is fractions or whole numbers, 'of' simply means 'multiplied by'.*

The children are also encouraged to play with fraction pieces. Using fraction pieces, we can go on to sums like 1/2 times 1/4, 1/2 times 1/5, 1/3 times 1/2, 1/4 times 1/4 etc.

Alongside the work with fraction pieces, the children are asked to write the result like 1/ 2 times 1/5, 1/2 x1/5 = 1/10

Multiplication by Composite Fractions

This means problems like 2/3 times 6, 3/4 times 8 and so on. We need to get them to recall composite fractions by asking different questions.

Example:

Diya: What is 3/4 of 12?

Child: 9

Diya: Can we say 3/4 times 12 is 9?

Child: I did not understand

Diya: Okay. You know that 3/4 means 3 times 1/4

Child: Yes

Diya: What is 1/4 times 12?

Child: 3

Diya: Good! What is 3 times 3?

Child: 9

Diya: So what would be 3/4 times 12?

Child: Yes I got it. It is 9.

Diya: Very good! Now tell me how we can write it.

Child: 3/4 x 12 = 9

Diya: Now tell me – what process did you follow to get 9?

Child: Bhaiya, first I found 1/4 of 12, then multiplied the result by 3.

Diya: What is 2/3 times 1/2?

After taking sometime a child may say,"Bhaiya, 2/6."

Diya: Excellent! Can you explain how?

Child: Bhaiya, first I found 1/3 times 1/2 and that is 1/6. Then I multiplied by 2 and that is 2/6.

Give the children similar problems to solve and help their clarity by asking questions. When they become confident, write a few of their results on the blackboard, ask them to observe the pattern and find the process of multiplication.

Division of Fraction

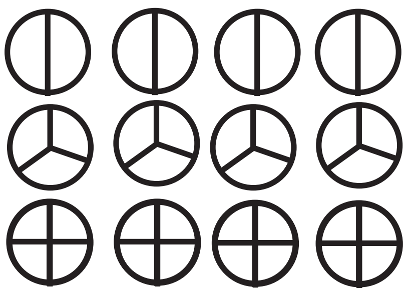
Many people (not just children!) find the division of fractions very confusing. In general we are all familiar with whole numbers. While dividing a whole number by another whole number, we always get a number that is less than the previous number. But in case of fractions we sometimes get a result that is GREATER than the number being divided! To understand the concept, let us go through some examples.

4 ÷ 4 = 1
4 ÷ 2 = 2
4 ÷ 1 = 4
4 ÷ 1/2 = 8
4 ÷ 1/4 = 16

To experience this we give 4 full circles and ask the children to divide them by 1/2, 1/3, 1/4, 1/6 and 1/8 and see how many parts or groups can be obtained from it. Note down the result on the board.

Dividing 4 full circles each into two halves will make 8 halves. Dividing into thirds will yield 12 pieces, into quarters will yield 16 quarter pieces, and into sixths should give 24 pieces, each one-sixth. And dividing into eighths will give 32 pieces, each one-eighth.

This is followed by an example like:



We have four rotis. If we share the roti by giving each person half a roti, how many people can we share it with? If we share the roti by giving each person a quarter, then how many people can we share it with? If we give each person 1/3 of a roti, then how many people will get it?

Note: As we have said earlier, this concept is confusing. The very real danger in introducing a method is that children will happily adopt it without fully understanding the underlying concept. Therefore it is very important that the above activity and similar examples are taken up many many times before proceeding to the next step below.

Diya: What is the pattern that you see in these sums?

The children may find it difficult to find the pattern. Now the teacher goes one step ahead and asks what is

$4 \div 1/2 = 4 \times 2$

$4 \div 1/3 = 4 \times 3$

$4 \div 1/4 = 4 \times 4$

$4 \div 1/6 = 4 \times 6$

4×2, 4×3, 4×4 and 4×6

Now the children may pick up the clue and start answering.

Child: The denominator of the fraction is multiplied with the number.

Diya: Very good. When we divide a whole number by a fraction, then the denominator of the fraction gets multiplied by the whole number given. Now come and see what happens when we divide a fraction by another fraction.

Suppose we have 1/2 roti. If one person eats 1/2, how many people can eat it?

Child: One person only.

Diya: If each one will eat one fourth roti, how many people can share it?

Child: Two

Diya: If we give one eighth to each one, then how many people can share it?

Child: Four people.

1/2 ÷ 1/2 = 1
1/2 ÷ 1/4= 2
1/2 ÷ 1/8= 4

Diya: Can you tell me what is happening here?

Child: Yes bhaiya, it is becoming half.

Diya: What do you mean?

Child: There are two halves in one, but there is only one half in a half. Also there are four quarters in a one whole, but there are only two quarters in a half. The number share becomes half.

Diya: Very good explanation. (Writing the relationship on the board) Can you find any relationship between the two denominators?

Child: The denominator of the second fraction is divided by the denominator of the first fraction.

Diya: Excellent! Now tell me if we have 3/2 roti and each person's share is 1/2, how many people can share it?

Child: 3 people.

Diya: How did you get it?

Child: There are 3 halves. When each one is eating one half, three people can eat.

Diya: If you give quarter roti to each, how many people can eat 3/2 roti?

Child: There are two quarters in one half, so there will be 6 quarters in 3/2. Therefore, 6 people can eat.

Diya (writing on the board):

3/ 2 ÷ 1/2= 3

3/ 2 ÷ 1/4= 6

See what happens when the denominator of the divisor is multiplied by the dividend.

Example 1: 3/2 ÷ 1/2
= 3/2 x 2/1= 6/2=3

Example 2: 3/2 ÷ 1/4
= 3/2 x 4/1= 12/2= 6

Remember, when you divide a fraction by another fraction, it is the same as flipping the divisor and multiplying by the dividend.

Example 3: 3/4 ÷ 2/3 = 3/4 x 3/2 = 9/8

Example 4: 5/6 ÷ 3/4 = 5/6 x 4/3 = 20/18 = 10/9

Fractions are introduced in higher classes in mirambika and does not, strictly speaking, fall within the ambit of Initial Mathematics. However, this topic has been taken up with children innovatively by Jasbir bhaiya such that children learn and understand fractions easily. We have included this chapter here to give the reader a taste of how fractions can be approached creatively and effectively with children.

CHAPTER 10

Word Problems

The term "word problem" has been widely used in the parlance of school mathematics. In mirambika, however, it is not so much looked upon as a "problem" to be hammered out to arrive at a solution for its own sake. Rather, these are illustrations of a situation that life presents, either real or in a story form. The child does with these as she naturally must, negotiating with her whole being to make sense of the situation as part and parcel of the act of living. Using her faculties, she negotiates her way through, learning skills to deal with life and addressing challenges and solving problems in real-life situations. Therefore, in Initial Mathematics, throughout the age groups, we introduce real-life situations to the child and apply a mathematical operation. It is only then that we arrive at a mathematical expression.

The child solving the problem sees that the operation is just a way or a new language to express what he has already solved. It does not solve the problem. That is for the child. But the operation makes the same task easier to express and understand. When we construct word problems that are real-life situations, we keep the following factors in mind:

1. Closer to the child

The context of the problem should be close to the child. Like the toys the child plays with, materials the child uses, family members, games that he likes, celebrations he participates in, his circle of friends, age, height, weight and the child's interest area.

2. Educationally significant

The problem should include elements that would enhance the child's learning, suited to his capacity. It could integrate other knowledge that you may want the child to acquire and be related to the projects that the child is working on.

For example,
1. Distances between two cities
2. Length of rivers and height of mountains
3. Population of cities, states and countries
4. Number of trees in the school

3. Emotional involvement and intellectual challenge

The situations that we create should engage the child emotionally and at the same time, it should encourage him to think. Use whatever knowledge you have about the children in the class to create the situation.

Some useful hints are given below:

- Take a child's name in the word problem.
- Bring in the situation of some important events like, the child distributing sweets on her birthday.
- Bring in the situation of some important events like, the father

making cake for mother's birthday or the girl is breaking record in weight lifting.

- Give examples of their favourite games, like the boy scoring runs or points in his favourite game, cricket.
- Give the example of some good habits, like the boy going for morning walk with his grandfather.

4. Size of numbers

We should be aware of the size of the numbers we are using in problems. It should be relative to context, comfortable for the age group and mindful of the ability of the individual child.

5. Addressing values

You could address a lot of values through mathematics problems like:

- Sharing of things among family and friends.
- Celebrating birthdays of friends, aspiring for their growth.
- Creating awareness of Nature around us.
- Creating awareness of physical development.

Keywords in word problem

Often, a child solving a word problem looks for a keyword, which will help to solve the problem. The difficulty here is that the child does not put in the effort to understand the problem in its completeness. So we should not encourage a child to look for keywords.

When children find the word 'more' or 'total', they tend to add. Similarly, if they find the word less and left, they subtract. This becomes a habitual response and they draw a conclusion without visualising the whole problem. They may not be able to deal with multi-step problems or a problem having no keywords. Worse, the keywords that they are looking for may mislead them.

Example: After putting 4 more fruits in the basket, Soham counted that the total number of fruits in the basket was 12. So how many fruits were there in the basket earlier?

In the above example, if the child follows the keyword 'more' or 'total' without understanding the context, he will add the numbers instead of subtracting.

Type of problems related to addition and subtraction

1. Changing situation: This type of problem consists of three elements: initial value, change value and final value. Out of these three any two are given. The third is unknown and has to be found.

Example 1: There were 8 litchis in a basket. Today morning father collected

5 litchis from the tree and kept in the same basket. Now how many litchis are there in the basket?

Example 2: एक जंगल में चार (four) हाथी थे। एक दिन पानी पीने के लिए गए। रास्ते में उनको आठ (eight) हाथी मिले और उनके साथ दोस्त बन गए। फिर सबने मिलकर नदी में जा कर पानी पिया। बताओ, कुल कितने हाथी पानी पीने के लिए गए थे?

2. Combining Situations: The problem of this type is subtly different from the changing situation ones. Here also two elements are given and you have to find the third, but the language of the problem is different.

Example 1: One day Suhani and Aditi thought of decorating flowers on the table. First Aditi arranged 7 flowers and Suhani arranged 8 flowers. Can you tell how many flowers both of them have arranged on the table?

Example 2: रोहन और बबिता एक दिन बगीचे में आम का पौधा लगाने गए। बबिता ने सात (seven) पौधे लगाए और रोहन ने छः (six) उगाए। दोनों मिलकर कितने पौधे लगाए?

3. Equalise: In this type of problem, there is an initial value and a target to reach. How much do you need to add or subtract to reach that target?

Example 1: There are 18 children in the green group, but only 13 chairs to sit on. So how many more chairs do we need?

Example 2 : आप लोगों ने कुछ देवी और देवताओं की मूर्तियाँ बनाई। जितनी भी मूर्तियाँ बनाई उनमें से बारह (twelve) देवताओं की मूर्तियाँ हैं। सब मिलाकर सत्रह (seventeen) मूर्तियाँ हैं। अब बताइये कि उनमें से देवियों की कितनी मूर्तियाँ हैं?

Example 3:. चींटी मेहनती होती है। बड़े मज़े से काम वह करती है। दिन-भर खाना ढूंढती रहती है। एक दिन उसे मिले चीनी के पाँच (five) दाने। घर में लाकर रख दिया तो हो गए इक्कीस (twenty-one) दानें। तो बोलो-बोलो पहले उसके घर में थे कितने चीनी के दाने?

Example 4: मिलजुल कर हम रहेंगे, लड्डू बाँटकर खाएँगे, दीदी-भैया दस (ten) लड्डू लाए हैं, पर समूह में पंद्रह (fifteen)बच्चे आए हैं, तब भैया ने पूछा, "सुदर्शन बताइये, अगर सबको एक-एक लड्डू देंगें तो हमें और कितने लड्डू चाहिए?

4. Partitioning: Partitioning is an action of taking away some objects from a collection and finding the number of objects that remain.

Example 1: There were 12 rabbits in a house. There was not enough space for them in that house, 4 of them were taken to another house. How many stayed back in the old house?

Example 2: Maitreyee has seven toffees. She gave two toffees to Ameya. Now how many toffees does she have?

5. Reduction: Reduction is the process of finding out the number of objects taken away or removed, when the starting amount and the remaining amount are known.

Example 1: Rini collected 12 flowers and kept them in a basket. Risaa took some flowers to arrange as an offering to The Mother. Rini found 4 flowers remaining in the basket. How many flowers were offered to The Mother?

Example 2: Zici had 8 flowers. He distributed some flowers and had 5 left. So how many flowers did he distribute?

6. Comparison: To find the difference between two sets of objects. Basically to find out how much one set is more or less than the other set of objects.

Example 1: नीले समूह में अट्ठारह (eighteen) बच्चे हैं। लाल समूह में सोलह (sixteen) बच्चे हैं। अब बताओ, नीले समूह में लाल समूह से कितने ज़्यादा बच्चे हैं?

Example 2: विष्णु जी के हैं चार (four) हाथ, दुर्गा माता के हैं दस (ten) हाथ, अब बोलो विष्णु जी के दुर्गा माता से कितने कम हाथ हैं ?

Word Problems through Stories

Stories play a vital role in the growth and development of children. If the story is interesting and imaginative, children listen to the story with complete attention, spellbound. In mirambika, the children are introduced to the world of initial mathematics through stories.

It is interesting to know that through the process of storytelling, along with the development of initial mathematics, many vital aspects are also addressed and developed in the child such as

Listening skill

- attention span
- comprehension
- imagination
- visualisation
- remembering the numbers
- calculation

This process starts with children of the 5+ age group with short stories, in small groups of 5-6 children. The selection of the story depends upon the children's interest. The Diya normally asks the children to choose characters for the story. After a decision is made, the Diya creates the story on the spot, according to children's potential, need and interest. Counters like animal figures, colorful stones, sticks, marbles and seeds are used as counters and are kept in front of each child before starting the story.

The example below gives a glimpse of the process.

"Once there was a big-big-big forest. Many animals were in the forest. Children are then requested to name the animals.

Diya asks, "Tell me, how many animals are there in that forest?

The children can then arrange the toy animals or pointers in front of them and after children count, the story resumes.

"Do you know who was the king of the forest?" Children would immediately say "Lion!" The Diya uses voice modulation. "No, no! They had a different rule in that forest. Every day one rabbit used to be the king. The rabbit used to wear a golden crown and would sit on a big golden throne. Every Thursday, the king rabbit used to visit the jungle, sitting on an elephant. Three soldier elephants used to follow the rabbit king."

"Now tell, how many elephants used to go with the king in total?

And can you tell how many legs the four elephants would have in total?

Behind the elephants, in that procession, bears used to walk in two rows. In each row, four bears used to walk like courtiers of the king.

Now tell how many bears would there be?"

Like this, the Diya extends the story. It starts with simple addition, within five or ten numbers and understanding the children's capacity, gradually, after a few days, the Diya includes more operations in the story.

Initially, children ask the Diya to repeat the numbers. If it happens often, then the Diya discusses with the children and sets a condition that the numbers will be told twice only and not more than that. Then children put an effort to listen attentively.

When children get used to doing four operations in one story within ten numbers, the Diya increases the number up to fifteen or twenty according to the children's potential. By the end of the year, a few children start doing the operation mentally whereas a few still need objects to calculate.

With six and seven-year olds, the story can be longer. It is always better to do this in small groups of 5-6 children. Initially, children do the operation one step at a time and the story proceeds further. The Diyas have to be vigilant to see what is happening with each child. If the child is not comfortable, then it is better to do it in a different way or with objects or through a short story with simple operations.

In each group there are always three or four children who may be very quick in doing calculations. In this case, these children can be told the story in full, at one go. This activity can be carried out in a flexible manner according to the situation and needs of the child or children.

Question 1: हाथियों की सैर

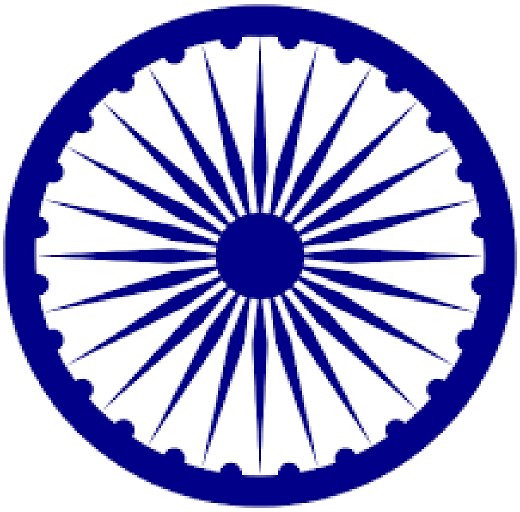
एक घने जंगल में बहुत सारे हाथी रहते थे। एक दिन सात (seven) हाथियों ने सोचा कि चलो घूमने चलते हैं। बहुत दूर चलने के बाद दो हाथी जामुन खाने के लिए रुक गए। बाकी हाथी चलते-चलते एक नदी के पास पहुँचे । वहाँ उनको पांच (five) हाथी मिले। फिर थोड़ी देर बाद आठ (eight) हाथी और मिले। वह सारे भी उनके साथ घूमने के लिए चल पड़े। वे चलते-चलते मीराम्बिका के फाटक के पास पहुँचे और चौकीदार भैया से पूछा "ये जो तीन (three) तरफ रास्ते गए है; ये कहाँ-कहाँ गए है?" चौकीदार भैया ने उन्हें तीनों रास्ते दिखाकर कहा- एक रास्ता जंगल-जिम की तरफ गया है, एक जूनियर पार्क की तरफ गया है और एक रास्ता मीराम्बिका की तरफ गया है। फिर हाथियों ने सोचा चलो, हम सब तीन (three) हिस्सों में बट जाते हैं और तीनों जगह देख लेते हैं। फिर जितने हाथी जंगल-जिम गए उतने हाथी जूनियर पार्क गए और उतने ही हाथी मीराम्बिका भी गए। अब बताइए कितने हाथी जंगल-जिम गए, कितने हाथी जूनियर पार्क गए और कितने हाथी मीराम्बिका गए?

Question 2: अशोक चक्र

Topic: India's Independence Day

• हमारे भारत के झंडे में जो बीच में एक चक्र है। उस चक्र को हम अशोक चक्र कहते हैं। इस चक्र के बीच में बहुत सारी

तिल्लियाँ भी हैं। अगर हम इन तिल्लियों को गिने तो कुल मिलाकर कितनी तिल्लियाँ होंगी ? अगर इन सारी तिल्लियों को हम दो (two), चार (four), छह (six) ऐसे गिनेंगे तो कितनी बार हमें दो (two) को गिनना होगा?



• उन तिल्लियों को अगर हम चार (four) हिस्सों में बाँटेंगे तो एक के हिस्से में कितनी-कितनी तीिल्लियाँ आएँगी? छः (Six) हिस्सों में बाँटेंगे तो एक के हिस्से में कितनी-कितनी तीिल्लियाँ आएँगी? आठ (eight) हिस्सों में बाँटेंगे तो एक के हिस्से में कितनी-कितनी तीिल्लियाँ आएँगी? बारह (twelve) हिस्सों में बाँटेंगे तो एक के हिस्से में कितनी-कितनी तीिल्लियाँ आएँगी?

Question 3: फूलों की रंगोली

Topic: Colour

एक दिन की बात है, मीराम्बिका में सभी समूहों के बच्चे मिलकर रंगोली बना रहे थे। रंगोली बनाने के लिए लाल समूह के बच्चे मिनति दीदी के साथ बाग में गए और लाल रंग के दस (ten) फूल लेकर आए। इसी तरह नीले समूह के बच्चे अपनी दीदी के साथ जंगल-जिम गए और नीले रंग के बारह (twelve) फूल लेकर आए। हरा समूह भी अपनी सुकान्ती दीदी के साथ बाग में गया और हरे रंग के तेरह (thirteen) पत्ते लेकर आया। अब बारी आयी पीले समूह की, सभी बच्चे रश्मिता दीदी के साथ बरगद के पेड़ के पास गए, उन्होंने देखा कि पेड़ के आस-पास बहुत सारे पीले रंग के फूल हैं। पीला समूह वहाँ से पीले रंग के नौ (nine) फूल लेकर आया।

• अच्छा बच्चों बताओ, अगर लाल समूह और पीला समूह अपने-अपने फूलों को मिलाएगा तो कुल कितने फूल होंगे?

• यदि नीले फूल और हरे समूह के पत्तों को एक साथ एक ही टोकरी में रख दिया जाए तो टोकरी के अंदर कुल कितने फूल और पत्तें होंगे?

•अगर सारे ही समूह के फूलों और पत्तों को मिला दिया जाए तो कुल कितने फूल और पत्तें होंगे?

फिर सभी समूह के बच्चो ने अपने-अपने फूलों को और पत्तों को धोया और एक टोकरी में रख दिया।

थोड़ी देर बाद वह बड़े समूह के एक भैया आए, उन्हें माँ मीरा के लिए एक माला बनानी थी तो, उन्होंने प्रत्येक समूह में से तीन-तीन फूल और हरे समूह से तीन पत्तियाँ ली।

• अब आप यह बताओ कि बड़े समूह के भैया ने पत्तों को छोड़ कर कुल कितने फूलों के साथ माला बनाई और प्रत्येक समूह के पास कितने फूल बचे?

Question 4: हिरण चले पहाड़ घूमने

एक जंगल था। उसमें बहुत सारे हिरण रहते थे। एक दिन चार (four) हिरण घास खाने के लिए एक पहाड़ के पास गए। पहाड़ बहुत दूर था। लेकिन वहाँ पर घास का एक बड़ा मैदान था। बहुत दूर चलने के बाद सब हिरण एक नदी के पास पहुँचे। दो (two) हिरण बोले कि हम यहाँ नदी में खेलेंगे। पहाड़ के पास नहीं जाएँगे। तो वह दो (two) हिरण वहाँ पर रुक गए। बाकी सारे हिरण उनको छोड़ कर आगे बढ़े। नदी के दूसरे किनारे उनको पाँच (five) हिरण मिले। फिर थोड़ी देर बाद सात (seven) और हिरण मिले। वह सारे भी उनके साथ घास खाने के लिए पहाड़ की ओर चल पड़े। चलते-चलते रास्ते में उन्होंने एक अनार का बगीचा देखा। बताइए कितने हिरण अनार के बगीचे के पास पहुंचे ?

जितने हिरण वहाँ पर थे, उतने ही अनार के पेड़ भी थे। जितने अनार पके थे उतने ही अनार कच्चे थे । हर पेड़ पर एक (one) अनार पक गया था । बताइए वहाँ पर कितने अनार के पेड़ थे? कितने अनार पक गए थे? कच्चे और पके अनार मिलकर कितने अनार हुए?

हिरणों ने मिलकर सारे अनार के पेड़ों से पूछा कि "क्या हम पके हुए अनार खा सकते हैं?" अनार के पेड़ों ने कहा-हाँ- हाँ, हम अपना सारा पका हुआ अनार नीचे गिरा देते हैं, आप उनको ले कर खा सकते हो । फिर पेड़ों ने सारे पके हुए अनार नीचे गिरा दिए । सारे हिरणों ने समान-समान बाँट कर अनार को खाया। कितने अनार नीचे गिरे और हर एक हिरण को कितने-कितने अनार मिले?

अनार बहुत मीठे थे । अनार खा कर सारे हिरण अनार के पेड़ों को 'धन्यवाद' देकर आगे बढ़े । अनार खा कर उनको बहुत शक्ति मिल गयी थी । सारे हिरण दौड़-दौड़ के पहाड़ के पास पहुँच गए । वह बहुत सुन्दर जगह थी । चारो तरफ हरियाली छाई हुई थी । बहुत सारे फूल भी खिले हुए थे । वहां पर चार (four) पंखुड़ियों वाले चार (four) तरह के पीले फूल थे, पाँच (five) तरह के तीन (three) पंखुड़ियों वाले लाल-लाल फूल थे और आठ (eight) तरह के दो (two) पंखुड़ियों वाले नीले-नीले फूल थे । वहां पर सब मिला कर कितने फूलों के पेड़ थे? पीले फूलों की पंखुड़ियों की संख्या कितनी थी? लाल फूलों की पंखुड़ियों की संख्या कितनी थी? नीले फूलों की पंखुड़ियों की संख्या कितनी थी? अगर सारे पंखुड़ियों को मिलाएंगे तो कितनी पंखुड़ियों हो जाएँगी?

सारे हिरणों ने पहले उस जगह पर छुपन-छुपाई खेली क्योंकि वह जगह बहुत बड़ी थी । इसलिए नौं (nine) हिरण आँखे बंद करते थे और बाकी सब छुपते थे । कितने हिरण आँखे बंद करते थे और कितने हिरण छुपते थे?

बहुत देर तक खेलने के बाद उन्होंने खूब सारी घास खाई और वापस अपने जंगल में आ गए । आते समय नदी के पास जितने हिरण रुक गए थे वे उनको मिले । फिर सबने मिलकर पानी में खेला और खूब मज़ा किया । पानी में कितने हिरण खेल रहे थे?

Question 5: चिड़ियों और दानों की कहानी

एक दिन की बात है, तीन (three) चिड़ियाँ दाना चुंगने के लिए अपने घोसलें से बाहर निकलीं । उन्होंने इधर-उधर देखा पर उन्हें दाना कहीं दिखाई नहीं दिया, वे ओर आगे बढ़ीं । थोड़ी ही देर के बाद अचानक उन्हें दस (ten) दाने दिखाई दिए, तीनों दानों के पास गईं और दानों को देखकर खुश हुई ।

उन्होंने दाने को आपस में इस प्रकार बाँटा कि एक चिड़िया के पास एक दाना ज़्यादा आया । फिर वे आगे बढ़ीं, चलते--चलते वे थोड़ा थक गईं, उन्होंने सोचा कि सामने जो घना वृक्ष है उसकी छाया में थोड़ी देर बैठेंगे व दुबारा घर की ओर प्रस्थान करेंगे । जैसे ही वे पेड़ की छाया में पहुँचीं तो वे और ज़्यादा खुश हो गईं क्योंकि उन्हें वहां और आठ (eight) दाने मिले । बहुत ख़ुशी के साथ उन्होंने उन दानों को लिया व आपस में इस प्रकार बाँटा कि अब तीनों के पास सभी दाने मिलाकर समान संख्या में थे । आराम के बाद तीनों घर की ओर चल दिए। जैसे ही वे घर के समीप पहुँचे तो उन्हें उनके मित्र मिले । सभी मित्रों ने एक दूसरे से नमस्कार किया ।

अब सभी मित्रों ने उन दानों को आपस में फिर से बाँट लिया। अब सभी के हिस्से में दो-दो दाने आए । बताओ उनको उनके कितने मित्र मिले होंगे ?

इसके बाद सभी चिड़ियाँ अपने-अपने घोसलें में चलीं गईं।

Question 6: रंगों वाली परियों की कहानी

Topic: Colour

एक बहुत सुन्दर जादुई दुनिया थी । उस जादुई दुनिया में बहुत सारे रंगो की रंग-बिरंगी परियाँ रहती थी । प्रत्येक परी का रंग एक दूसरे से भिन्न था, और आपको पता है सभी परियाँ अपने-अपने पेड़ों पर रहती थी ।

सबसे मज़ेदार चीज़ यह थी कि, जिस पेड़ पर जिस रंग की परी रहती थी उस पेड़ की पत्तियों का रंग उसी परी की तरह था । जैसा कि अगर लाल रंग की परी है तो जिस पेड़ पर वह रहती थी, उस पेड़ की सभी पत्तियों का रंग भी लाल था ।

अचानक एक दिन हवा चली और हवा धीरे-धीरे बढ़ती चली गई। इसी दौरान लाल परी के पेड़ से सात (seven) लाल रंग के पत्ते नीचे गिरे। नीली परी के पेड़ से ग्यारह (eleven) नीले रंग की पत्तियाँ गिरी, हरे रंग वाले पेड़ से बीस (twenty) हरी पत्तियाँ गिरी, गुलाबी से आठ (eight) पत्तियाँ, नारंगी से चौदह (fourteen) पत्तियाँ गिरी।

बताओ कुल मिलाकर गिरी हुई पत्तियों की संख्या कितनी थी?

थोड़ी देर बाद वहाँ दो इल्ली (caterpillar) आए। दोनों ने ज़मीन पर गिरी हुई पत्तियों में से पत्तियाँ खाईं, एक ने चार (four) लाल रंग की पत्तियाँ खाईं और पूरी लाल रंग की हो गई। दूसरी ने छः (six) हरी पत्तियाँ खाईं और वो हरी रंग की हो गई।

अच्छा बताओ, ज़मीन पर कितनी लाल पत्तियाँ और कितनी हरी पत्तियाँ बची?

फिर धीरे-धीरे हवा बढ़ी। जैसे ही हवा बढ़ी, तो लाल परी के पेड़ से और पत्ते गिरे। एक हवा के झोंकें से चार (four) पत्तियाँ गिर जाती थी। इसी तरह चार (four) बार तेज़ हवा का झौका आया और हर बार चार/चार (four) पत्तियाँ नीचे गिरीं।

बताओ बच्चों ! इस बार ऊपर से कितनी लाल रंग की पत्तियाँ गिरी? और अगर उनको पहले ज़मीन पर गिरे हुए लाल पत्तियों के साथ मिलाएँगे तो सब मिला कर कितनी लाल रंग की पत्तियाँ होंगी?

अब जैसे-जैसे हवा चली तो पत्तियाँ भी बढ़ने लगी। हवा के साथ पत्तियाँ उड़-उड़ कर एक नदी के पास तक पहुँच गयी। उस नदी में तीन कछुए रहते थे। जब तीनों ने इतनी सारी रंग-बिरंगी पत्तियाँ देखी तो बहुत खुश हुए। तीनों ने पत्तियों को इकट्ठा किया और सभी पत्तियों को मिलाकर आपस में इस प्रकार बाँटा कि सभी को समान संख्या में रंग-बिरंगी पत्तियाँ मिल गई। तीनों ने उन पत्तियों से अपने घर को सुन्दर से सजाया।

अच्छा बच्चों ! बताओ कि हर एक कछुए को कितनी-कितनी पत्तियाँ मिली होंगी?

Question 7: मानचित्र

Topic: Map

When Vivek bhaiya, Jyoti didi and Tilottama didi got the road maps to their home from each child, they decided to go to all the children's houses. They received Niraj's map first, so they decided to go to Niraj's house first. From Niraj's house they went to Kanishka's house, then they went to Sanvi, Arpan, Ona, Nitya and Pranab's houses. They started from mirambika at 9am. They spent 1 hour in each child's house. They spent total of two hours in travelling following each child's map. So, after how many hours and at what time did they come back to mirambika?

Question 8: मानचित्र

Topic: Map

Kabir made a map to a far away tree. Orange group children decided to go there. They spent two days in the land of party, one day in the owl's house, three days in the land of slides, one week in the land of colors, five days in the land of water, six days in the land of secrets and two days in the land of bouncing. After that, they reached the far away tree. How many days did they take to reach the far away tree?

Question 9: मानचित्र

Topic: Map

नीरज को भारत मानचित्र को देखने के बाद भारत भ्रमण करने का मन हुआ। वह अपनी मम्मा-पापा के साथ जम्मू कश्मीर गया। वहाँ उन्होंने हिमालय पर्वत और अलग-अलग नदियों को देखा। वहाँ पाँच दिन रहने के बाद वे लोग पंजाब गए। वहाँ पर उन्होंने अमृतसर का गुरुद्वारा भी देखा। पंजाब में वे लोग तीन दिन ठहरें। फिर पंजाब से राजस्थान जाने में उन्हें दो दिन लग गए। राजस्थान में ऊंट की सवारी की और बहुत सारे राजा महाराजाओं के किले में घूमें। नीरज ने जितने दिन जम्मू-कश्मीर में बिताए थे, वे राजस्थान में दो दिन कम ठहरें। क्या आप बता सकते हो, कि वे राजस्थान में कितने दिन ठहरें? राजस्थान से मध्यप्रदेश आने में उन्हें एक दिन का समय लगा। मध्यप्रदेश में नीरज को tiger national park देख कर ख़ूब आनंद आया। राजस्थान में जितने दिन ठहरें थे, मध्यप्रदेश में उससे दो गुना अधिक समय बिताया। क्या आप बता सकते हो कि नीरज मध्य-प्रदेश में कितने दिन ठहरें? मध्यप्रदेश से वह कर्नाटक गए। कर्नाटक जाने में उन्हें दो दिन लगा। कर्नाटक में उनको मुकुंद मिला। मुकुंद ने नीरज को सारी जगहों को सुंदर से घुमा कर दिखाया। जितने दिन जम्मू कश्मीर में रूके थे, उससे अधिक दो दिन समय कर्नाटक में ठहरें। क्या आप बता सकते हो, कि नीरज कर्नाटक में कितने दिन ठहरें? कर्नाटक से वे केरल गए। केरल जाने में उन्हें एक दिन लगा। केरल में वे नाव में बैठ कर जंगल देखने गए जो कि पानी के बीच में था। केरल में वे दो दिन ठहरें। केरल से वो ओडिश गए। ओडिशा जाने में उनको दो दिन का समय लगा। ओड़िशा में उन्होंने बहुत सारे मंदिर देखे। केरल में जितने दिन थे उससे एक दिन अधिक समय ओड़िशा में ठहरें। ओड़िशा से नीरज को अरुणाचलप्रदेश जाने में चार दिन का समय लगा। ओडिशा में जितने दिन ठहरें थे उतने ही दिन अरुणाचल प्रदेश में ठहरें। अरुणाचल प्रदेश से वे दिल्ली वापस आ गए। दिल्ली आने में उन्हें तीन दिन का समय लगा। अब बताओ कि नीरज कौन-कौन से जगह में कितने दिन ठहरा? उसने किन जगहों में सबसे अधिक दिन बिताए, किस जगह में सबसे कम दिन बिताए? ये उसका कितने दिन की भ्रमण था?

Question 10: हुप्पा टिड्डा और उसके दोस्त

Topic: Little creatures in our surrounding

एक दिन हुप्पा टिड्डा और उसके कुछ दोस्त गुल्ली-डंडा खेल रहे थे। खेल खेलने के बाद सभी लोग चल पड़े तालाब की ओर पानी पीने के लिए। तालाब जाकर हुप्पा टिड्डा और उसके दोस्तों ने जी भरकर पानी पीया। पानी पीने के बाद उन सभी ने सोचा कि चलो क्यों न इस ठण्डे-ठण्डे पानी में नहाया जाए। हुप्पा टिड्डा और उसके दोस्त, सभी ने एक साथ मिल कर उस तालाब में छलाँग लगाई। छपाक-कक…….. छपाककक........!!!

छलाँग लगाने के बाद जैसे ही वे एक डुबकी के बाद ऊपर आए तो हुप्पा टिड्डा और उसके दोस्त आश्चर्यचकित रह गए। पता है क्यों?

क्योंकि जैसे ही वे ऊपर उठे तो उन्होंने अपने आप को बहुत सारे टिड्डो के बीच में पाया। ये टिड्डे उस तालाब में पहले से ही थे।

अब अगर तालाब में टिड्डो को गिना जाए तो कुल मिलाकर छब्बीस (twenty six) टिड्डे हैं। अठारह (eighteen) टिड्डे पानी में पहले से ही थे और हुप्पा टिड्डा को मिलाया तो उन्नीस (nineteen) टिड्डे हो गए।

• ज़रा सोचकर बताओ बच्चों कि, हुप्पा टिड्डा के साथ उसके कितने दोस्त आए थे?

हुप्पा टिड्डा और उसके दोस्तों ने मिलकर बाकी टिड्डो से पूछा कि आप लोग यहाँ कब आए? फिर टिड्डो ने बताया कि हम लोग इस तालाब में थोड़े गहरे पानी के अंदर ध्यान लगा रहे थे लेकिन, हमें कुछ छप-छप की आवाज़ आई तो हम ऊपर आ गए, यह देखने के लिए कि क्या हुआ? पर यहाँ तो सब ठीक है तो हम वापस से नीचे जा रहे हैं, ध्यान करने के लिए।

तभी हुप्पा टिड्डा के एक दोस्त ने पूछा कि, “आप लोग कैसे ध्यान लगाते हो, क्या हम देख सकते हैं ?”

दूसरे टिड्डो ने कहा, "हाँ हाँ क्यों नहीं, पर आपको उसके लिए शांति के साथ नीचे जाना होगा।"

फिर सभी टिड्डे मिलकर नीचे गए। जब वे नीचे पहुँचे तो उन्होंने देखा कि छः (six) पंक्तियों में बहुत सारे टिड्डे हैं, और सभी ध्यान लगा रहे हैं। हर एक पंक्ति में चार-चार (four-four) टिड्डे बैठे हुए थे।

• क्या आप बता सकते हो, कि नीचे गहरे पानी में कुल कितने टिड्डे बैठकर ध्यान लगा रहे थे?

फिर ऊपर से आए अठारह (eighteen) टिड्डे, हुप्पा टिड्डा और उसके दोस्त सभी ने मिलकर ध्यान लगाया। थोड़े समय के बाद ध्यान लगाने का समय समाप्त हो गया।

सभी टिड्डे अपने-अपने घर की ओर चल दिए। हुप्पा टिड्डा और उसके दोस्तों ने देखा कि सभी टिड्डो ने अपने आपको दो (two) समूहों में बाँट लिया है और अपने - अपने घर की ओर जा रहे हैं।

• आपके हिसाब से एक समूह के कितने-कितने टिड्डे आए होंगे?

फिर हुप्पा टिड्डा और उसके दोस्त भी अपने-अपने घर चले गए। घर जाकर उन्होंने अपने परिवार और दोस्तों को बताया कि ध्यान कैसे लगाते हैं।

प्यारे बच्चों,

क्या आप बताओगे कि सभी टिड्डो ने गहरे ठंडे पानी के अंदर कैसे ध्यान लगाया होगा? आप जब ध्यान लगाते हो तो आप के मन में किसका चित्र आता है? चित्र बनाकर लिखो।

The universe is not merely a mathematical formula for working out the relation of certain mental abstractions called numbers and principles to arrive in the end at a zero or a void unit, neither is it merely a physical operation embodying certain equations of forces. It is the delight of a Self-lover, the play of a Child, the endless self-multiplication of a Poet intoxicated with the rapture of His own power of endless creation.

- Sri Aurobindo

CWSA-Essays in philosophy and Yoga-201

CHAPTER 11

Projects

The growing child wants to deal with the real world like a grown-up, but this is not always safe or feasible. Mathematical Projects help bridge this gap, enabling children to experience real-life situations. Concepts like Pattern and design, Time, Money, Measurement, Banking, and Market are ideal for projects, but a resourceful teacher can think of more. During projects, concepts are introduced through a range of activities at the children's level. The learning is practical and, like life, not restricted to any one subject. Here are some examples of projects undertaken in different groups in mirambika:

Observing the time measured by sand clock

Project: Time

Green Group, Age 5+

At this age, children live in eternity. As the mind develops, they start observing, experiencing and exploring the rhythms of the universe – the rising and setting of the sun, the changing of seasons, weather, the blossoming of flowers, plants and the growth of body and mind. Questions arise in the mind. As this vast knowledge is absorbed, the awareness of time emerges. Alongside the importance of time, another concept takes root – timelessness.

Since Time is both universal and fundamental, an astonishing range of activities is available for children to experience the concept like shadows – observing and playing with them, measuring shadows at different times, the movement of the sun, observing sunrise, sunset, day and night, week and month, making different clocks like water clock, sand clock and sundial, observing the clock and learning to tell time, making the time chart, observing growth in Nature.

Children express their understanding according to their age, using pictures, stories and poems. Here is a detailed plan of what is practiced in mirambika.

Aims: To experience 'time' in the child's own life as well as in the surroundings

Developing observation

- Observing movement of the sun at intervals from sunrise to sunset and marking the shadow size standing in one point, making a sundial
- Learning to specify directions (play way method - face towards east, run to the south, likewise)
- Observing own shadow at different points of time and noting changes in direction and size
- Observing Nature closely at different times
- Sprouting different seeds and seeing their growth rate
- Reading time from an analog clock (a clock with hands)
- Noting their own arrival time at school and making a chart
- Observing how much time they take to finish lunch or any work
- Observing the duration – 1 minute, 2 minutes
- Measurability – Using instruments such as a sand clock to measure the rate of activity: sorting shapes, coins, threading beads. Sample investigations: How many beads can you thread in a given time? How long does it take to thread 20 beads?
- Making own daily routine chart in a small book form from the time of waking up to bed-time.
- Making own growth book with photographs and drawings from birth to the present.
- Calendar – Making a chart of birthdays in graphical form and with colours

Thinking

- Discussion on 'what is time', 'why is time necessary', 'can one see time', 'can one touch time','if there is no watch is there time or not', 'if time will not be there, what would happen'
- Relating to time sequentially, for example, routine activiites like juice time, games time, music time, work time and lunch time
- Concept of slow-fast, early-late, past-present-future, week, month, year, hour, minute and second
- Evolution of clock such as shadow clock, sand clock, water clock, sun dial, candle clock, pendulum clock, electronic clock and digital clock
- Making different types of clock like sand clock, water clock and sun dial

Reflective questions

- Time goes fast and time goes slow. When you make something with interest and somebody says, 'time is up', how do you feel?
- When do you feel time seems to pass slow and when do you feel time seems to pass fast? For example, you wait for your father to come and take you to your favourite place and when you are doing your favourite activity.
- What are the things we do slowly and what are the things we do fast?
- When and where do we do things slowly and when and where we do things fast?

Language development

Development of writing, reading, imagination, memory, expression ability

- Hindi language vocabulary: समय, छाया, सूरज, रात, दिन, सुबह, दोपहर, शाम, ऋतु, साल, महीना, हफ्ता
- Point in time, moment, now, duration, span, succession continuity, describing events sequentially
- Poem making on समय, परछाई
- Worksheets on
  a. Correct the sentence
  b. Names of different days and months
  c. Jumbled words (what comes under what day, month and season)
- Comparison between day and night, summer and winter (experessing the differences through drawing and writing)
- Learning songs on साल, महीना, हफ्ता
- Imaginary writing on ' अगर घड़ी समय नहीं होता तो क्या

Initial Mathematics

- Learning to see patterns in shadows (relating with size, height, and angle)
- Learning to see the daily pattern of their life
- Learning to read the sun's direction
- Questions related to days and months like '2 days after Sunday, 3 months after July'
- Learning to see the pattern of the clock (minute, hour and second)
- Learnig skip counting in fives
- Word problems. Sample: "We are going to a jungle for a picnic. The bus is supposed to come at 9am. But it came 15 minutes and 26 seconds late. At what time did the bus come?"
- Worksheet on clock, for example, Children are given blank clock faces with the time written below. They then draw the hour hand and the minute hand showing that time on the clock.
- Children make clock with cardboard, they play 'show me the time' game with Diyas and with one another

Hand Skill

- Learning to draw lines by using a ruler while making the sundial
- Shadow marking with chalk
- Making different types of clocks like sand clock, water clock and sun dial
- Drawing the pictures of fast and slow moving animals as well depicting day, night, seasons, one's own routine during the day
- Making clock with card board, making a big clock on the floor with sticks, leaves and other things from nature.
- Making shadow puppet show

Games time

- Playing 'touch the shadow' game
- Playing 'light and shadow' game
- Playing 'captain says' game (turn your face to the east, west, south, north, south-east)

- Change the place with days of the week, months of the year, seasons
- Playing 'What is the time, Mr. Wolf?'
- Walking like fast animals and slow moving animals
- Playing 'Fire on the mountain'- "those whose birthday falls on the 21st of February, stand together!"
- Game measuring the time-taken to run from one place to the other
- Playing a game on finding out the time it takes to stand in a yogasana posture

Developing awareness through games

- How long can you be really silent (in the body)?
- What helps you to stay silent
- Does your body say something to your mind or feelings when you sit silently?

Evaluation

Through questions and interactions:

- What did you learn while doing all these activities?
- Why are we making different types of clocks?
- Why are we marking the shadow?
- What did you observe in the sundial?
- What did you observe in the sand clock?

Conclusion

One can never tire of looking at Creation. The vast night sky, with countless stars, the planets, and the moon arranged in perfect order… the patterns and designs of the plant kingdom expressing beauty in various ways, through birth, growth and death… and then its culmination, at least for now, in the expression of mind.

At this age, as faculties develop, the child's mind begins to take a complete shape. The child's senses begin to extend from the inward looking deep into the outer world. The inner-world in which there is so much fantasy and wonder begins to yield to the knowledge of the practical, or, as we call it, 'reality'. This then, is the challenge facing the Diyas - to create a bridge, between this wonderful inner world and the reality of the outer world so that the child traverses both the worlds, living in harmony with the time and space of both the worlds.

Project: Coin

Yellow Group, Age 6+

The Need: The topic 'Coin' emerged when we found that some children were buying things from shops themselves, while other children did not know the value or role of money. Some parents expressed concern.

We were aware that this had to be done at the level of 6-year-olds, with a lot of play. We tried to come up with ideas where they would enjoy exploring coins. In the group the children were learning to carry out the four operations with two-digit numbers mentally, do skip counting, reading and writing in English and Hindi. We tried to integrate all these aspects, as well as an all time favourite – Art. This was also an opportunity to help the children develop their concentration.

Concepts

- Identification of coins of different denominations
- Exchange of coins of one denomination with others
- One rupee is equal to 100 paise
- Conversion from rupees to paise and vice versa
- Carrying out operations with money
- Understanding the value of money

Note: This is, as usual, a list of activities, as they were carried out in mirambika. These are suggestions, and the facilitator is free to come up with his or her own ideas in addition to, or in place of these. Also, the described materials are not essential, and can easily be substituted with available materials. If, for example, old coins are not available, modern coins can be used. The mathematical principles will not change, and the learning can be equally engaging with the resources that you do have.

Activity 1: Classifying and counting coins

The children were given a handful of coins of different denominations. They were asked to separate them according to shape and size. Next, they tried to identify each coin of different denomination. They then counted the number of coins in each denomination.

Activity 2: Recognition of coins

Children tried to identify coins of different countries, including India.

Most children could recognise an Indian coin visually because of the Ashok Chakra. Some children who could read English tried to guess the country from the words in different languages. "Italy!" said one. "Europe!" said another.

Activity 3: Exploration by touch

The children were given Indian coins. They had to then close their eyes, feel the coin and make as many detailed observations as they could. The idea was to see if they could make detailed sensory observations.

The blindfolds made for a lot of excitement and laughter. The children came up with many observations. Most wanted to guess the identity of the coins. Some listened attentively to their friends' guesses and used those as clues for their own guesses.

Some knew that coins were of different sizes. So they would hold each coin in their hand and try to guess its identity

- Many said there was an Ashok Chakra on the coin
- Some said it was written in both Hindi and English
- Some were trying to feel the coin's value

- Some noticed the patterns or drawings
- Some talked about the Raja's head or 'Photo' on the coin
- Some were trying to identify the coin's value through its specific shape, size and weight

Activity 4: Exchanging coins

The children were asked to exchange coins of bigger denomination with small denomination. E**xamples:** one 10p = two 5p, one 20p = two 10p or four 5p, one 50p = ten 5p or five 10p, and so on.

Then they were asked to make different values with mixed denominations like

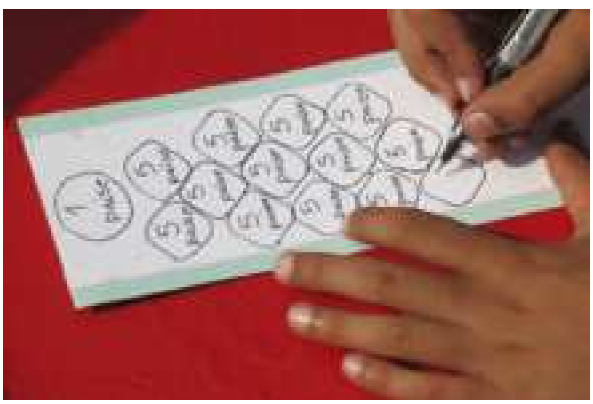
I. Make 50p using 5p, 10p and 20p coins

II. Make 75p with 5p, 10p, 20p coins

III. Make 100p with 5p, 10p, 20p and 25p coins or with 5p, 10p, 20p and 50p coins

Activity 5: Making cards showing one rupee with different denomination coins

Each child traced the coins on paper showing Re 1 = twenty 5-paisa coins, ten 10-paisa coins, five 20-paisa coins,



Tracing on paper five paisa coins

four 25-paisa coins and two 50-paisa coins.

Activity 6: Exchange of one rupee coin with different denominations

After playing with paisa coins we introduced the 1-rupee coin and asked how many paise made one rupee. Very few were able to say that 1 rupee was 100 paise. Then we started to exchange 1-rupee coins with different denomination coins. The children were happy to count with skips of 5, 10, 20, 25 and 50 to reach 100.

Activity 7: Setting up a bank and exchanging coins with each other

- To make the exchange of rupees for coins as real as possible for them, the children were asked to run a bank with coins. The Diyas exchanged coins with them. A Diya would give a rupee and ask for it to be changed into 20 paise coins. One child brought a fistful of coins. Diya said, "I do not want too much.

Cards of one rupee with different denominations

You should give exactly what I have asked for." She prompted the child to skip count in that denomination.

Cards of one rupee with different denominations

- Each child made a cue card for each denomination to follow whenever they needed. For example, draw or trace a rupee coin. On the same card, draw or trace twenty 5-paise coins. The child had to decorate it and write the bank's name, so that it could be displayed in the bank. (Cards created in Activity 5 can be used too.)
- When the children got confident in making 1 rupee with different denominations, Rs.2, Rs.5 and Rs.10 were introduced. They were asked to make a bigger value coin with different smaller denomination coins.
- The next step was the reverse of the previous one. A fistful of coins of a single denomination (say, 25p) was given to each child, with the question 'How many rupees is this?' Some came up with the idea of collecting coins in groups of 4, amounting to 1 rupee.
- Then a mixed denomination of coins was given to be counted. After making 1-rupee piles, they were asked how many paise and how many rupees there were in all.

Activity 8: Story sums

After conversion, they practiced the four operations through different story sums. Here is an example of a story sum:

One day, Shivani went to a shop nearby to purchase bread and a packet of milk. Her mother gave her fifteen ten paisa coins, five twenty paisa coins, twelve twenty five paisa coins and eight fifty paisa coins. Children, can you tell how much money she took with her? She gave half of her money to the first shopkeeper who gave her a loaf of bread. She brought back three ten paisa, two fifty paisa, four twenty paisa, and returned the money to her mother. Children, can you tell how much money she gave to the second shopkeeper to buy milk?

Initially the children were encouraged to add and subtract with the help of coins, then they did mental calculations.

Conclusion

This project on Coins gave children an opportunity to understand and develop a feel for money and what it is in physical terms. They became familiar with real coins and understood the play of numbers through coins. It gave them too an added confidence as they were traversing a little, the world of their parents who engage with money in their day-to-day lives. They developed a sense of importance and with that, a sense of responsibility while dealing with money.

Project: Measurement

Orange Group, Age 7+

This is a project that we generally do with 6+, 7+ and 8+ age group of children. At this age, the children start comparing their heights, practice counting steps, and play at measuring different things with their hand spans. They also measure the weight of different objects, and their own weights. This topic rides on children's natural interests, and becomes alive with daily activities of measurements, as children explore and experiment with length, height, distance, weights, volumes and capacities.

Measurement of Length

Aims

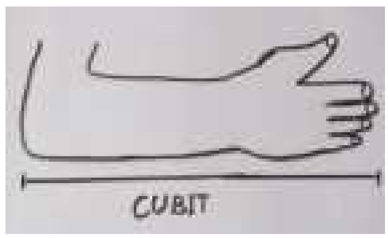
- Measure using one's own scale like their hands, feet, palm and fingers
- Compare one's own scale with others
- Measure by using sticks of different sizes
- Think about the necessity of one standard scale
- Be introduced to the metre scale and use of units
- Develop their capacity of estimation
- Develop fine and gross motor skills
- Develop interest, coordination and patience

Activity 1: Measuring body parts by using hand span and cubit.

The class started with a group discussion. The Diya asked who was the tallest in the group. The children started looking at each other and came up with many guesses. When asked to confirm who was right, they came up with a suggestion. All of them stood in a line and one child observed who was the tallest amongst them.

Diya: Can you tell me how tall you are without using a scale?

The children came up with the idea of measuring with their hand and arm. We had a discussion cum demonstration about these handy measuring instruments and children learnt about the hand span, cubit, finger's width and foot span. Then the children were encouraged to estimate their own height.



Each child came up with his own estimation and the Diya asked them to remember what they had estimated about their heights. They were then paired and asked to measure each other's heights. One child stood against the wall while the other marked his height with a piece of chalk on the wall.

Next they measured the height in hand spans. They then checked it against their estimation.

While measuring we found that some children were standing with their shoes on and their friends asked them to remove their shoes. A discussion took place and then the Diya asked them, "How many hand spans is your cubit?" The children easily measured it and came up with the observation that a cubit was two hand spans. The children were asked to measure the length of their leg, arm, middle portion of the body by using their cubit and hand span. They were asked to draw a picture of themselves and mention the length of each part on it.

Measuring each others height with hand span

Activity 2: Guessing and measuring different objects in the class room using hand span and cubit.

The children were asked to guess what would be the length of the table, width and height of the black board, and height of their cupboard in hand spans and write this in a tabular form.

To Measure	Measuring Tool	Guess	Accurate Measure
Length of table	Handspan		
Width of black-board	Handspan		
Height of black-board	Handspan		
Height of cupboard	Handspan		

After writing their estimations, they were asked to measure and write the accurate measurement in the corresponding column in the table.

Example:

Child 1: Bhaiya, when we measured the length of the table, I got 3 hand spans but Aadya got two and a half. How is that possible? (Other children also raised similar questions.)

Child 2 : All of our hands are not of the same length.

The questions that followed: So bhaiya, if all children have different measurements then which one will be right?

Bhaiya: All are right.

Child 3: So how can we say the length of the blackboard is six hands, six-and-a-half hands and so on?

Bhaiya: Yes, you are right. How can we solve this problem?

Child 4: We will take the longest hand.

Child 5: Why? We can take the shortest hand?

Others: No, we will take the medium length.

One child: Why can we not take a stick to measure?

The children had many questions after these activities.

Activity 3: Measuring a short distance using foot span

The children were asked to estimate the length of the classroom in foot spans and write it in a table. After writing they measured using foot spans.

Measuring a distance with foot spans

To measure	Measuring Tool	Guess	Accurate Measure
Length of the class-room	Foot span		
Any distance in your environ-ment	Foot span		

After measuring and observing each other's data they found out that the number of hand spans and foot spans varied.

Diya: Who has the maximum number of steps?

One child stepped forward.

Diya: Why do you have the maximum?

Child 2: Bhaiya, her footsteps are small. That is why she has more.

Note: From the above two activities, the children understood that if we measured with a short length, the measured units will be more and if we measured with a long length, the measured units will be less.

The children had many questions after these activities.

Activity 4: Measuring length at different places by using a standard stick

Children were paired. They were asked to first guess the distance from the class doorway to some point a short distance away. They noted this down in a table like the one below. Then they were given a stick and a piece of chalk and asked to measure it properly. This too was noted in the table as follows:

To measure	Measuring Tool	Guess	Accurate Measure
Class door to tree	Stick		
Any similar distance	Stick		

Note: While measuring, the Diya checked that they were putting the chalk mark at the end of the stick while placing the stick correctly on the previous chalk mark.

The children were happy that all had the same measurement. A long discussion followed about how the measurements were different when they were using hand spans and foot spans. They concluded that we needed a fixed measuring tool which would be common for everybody. The discussion continued.

Diya: We measured with this stick which gave us a uniform result. But if another group were to measure, they may not use the same stick that we used, so their results will be different. How can we then define a common measurement tool?

The children sat in silence and some started thinking.

Activity 5: Introducing the measuring scale and measuring tape

Diya: (Showing a one-metre measuring scale) Do you know what is this?

Some children: "It is a scale. I have one at my home. My sister has a small scale."

Diya: Yes, it is a measuring scale, also called a metre scale. This is a standard measure, used all over the world to measure length. The Diya passed the scale for all to see the markings on it.

Activity 6: Each child made an accurate 30cm scale

The children, in order to experience the units of measurement and to increase concentration span and to sensitise them to precision, were guided to make a 30cm scale with paper strip.

First, each child was given a wooden scale to observe the markings. This was followed by a discussion on what they had observed and what markings they had seen on the scale.

Measuring the length of the ledge with a self-made meter-scale

Diya: The questions we asked the children:

- What different types of markings do you see on the scale?
- How many long and small lines do you observe?
- What patterns do you observe?
- How many small lines are made between two longer lines?

- How many numbers do you read on the scale?
- What do ‘c’ and ‘m’ mean?

Through this discussion, the terms ‘centimetre’ and ‘millimetre’ were introduced.

To show them how it is done, the Diya began making her model scale by marking the dots for centimetres, and then drew the lines for them. Children were given a thick paper strip of 50cm each. After observing the Diya, the children also started marking their scales. With 2-3 attempts, each child made an accurate scale for themselves.

Activity 7: Measuring things in the classroom with the help of a self-made metre-scale

To begin this activity, we measured the length, breadth, height and thickness of one desk. Then we asked the children to measure whatever they could see in the classroom and record it in their notebooks. They measured a notebook, a table and the classroom. They recorded their measurements in their notebooks in a tabular form.

Name of the object	Length
Notebook	
Table	
Classroom	

Note: While measuring, children needed to be guided to place the scale precisely and mark correctly. We also guided them to write the units correctly.

Activity 8: Discussion of results

- The length of the table is 120 centimetres. How do you say that using metres?
- The length of the classroom is 7 metres and 35 centimetres. How many centimetres does this add up to?

Questions like this help clarify conversion from metres to centimetres and vice versa. Keep the meter scale handy for further reference in the course of the duscissuion.

Activity 9: Worksheet on conversion from centimetres to meters and vice versa

1) Write in metres and centimetres.

(m- metre, cm- centimetre)

Example: 125cm = 1m and 25cm

- 145cm =…… and……..
- 182cm =……..and ……..
- 256cm =……..and……..
- 474cm =……..and ……..
- 754cm =……..and……..

2) Write in centimetres

Example: 1m and 76cm = 176cm

- 1m and 54cm =..............
- 4m and 34cm =..............
- 5m and 78cm =..............
- 8m and 23cm =..............

- 6m and 96cm =...............
- One centimetre =.......... millimetre
- One metre = ... millimetre
- Make a metre scale of length 50 centimetre.

Activity 10: Making a crossword puzzle game board using the scale.

The children started with a cardboard sheet of dimensions at least 40cm × 40 cm. They had to make a 30cm × 30cm square. Using a 30cm scale, this then had to be divided into 10 rows and 10 columns, so that each small square was 3cm × 3cm. They then used this grid to create a crossword puzzle.

Activity 11: Measuring the running track with a meter tape.

This activity was done with 7-year old children. By this time, the children were used to measuring with a scale. They understood the terms, 'centimetre', 'millimetre' and 'meter'. It was time to get them to measure in meters. We cut a long satin ribbon rolled into one meter strips. In pairs the children measured the whole running track. They were in a hurry to measure and hence while turning the ribbon to measure using the next tape, they would have moved ahead by two or three centimetres. They therefore had to be guided to be precise. This exercise encouraged coordination, accuracy and patience while repeating the activity over a 400 meter track.

Measurement of Weight

Aims and Objectives

1. To develop a sense of heavy and light
2. To know the weight of objects the children use everyday – bag, water bottle, cricket bat, football, basketball and books.
3. To be able to guess the weight of objects.
4. To develop the skill of weighing with the use of hand balance.
5. To know standard measurement units and their conversion.

Activity 1: Estimating heavy and light objects (comparing two objects by weight and then measuring with a simple balance))

Requirements

One or more balances. Objects like bags, ball, bat, books, water bottles and stones.

Diya showed two different stones of clearly different weights and asked the children, "Which one is heavier?" The children checked one by one and guessed which was the heavier stone. After this, the Diya asked one child to put the two stones on either side of the balance and check who was right. This was repeated with several pairs of objects, to estimate the heavier of the two. By careful selection of objects, the difference in weight was progressively narrowed till it became challenging to estimate (accordingly to children's level).

Children used the balance to find out whose bag was heavier, whose water bottle was lighter. They then used their own creativity and curiosity to set up more tests. Each child estimated the weight of their object and placed the object on the weighing scale. This became a guessing game and was fun.

At the end of the day we asked the children to make their weighing scales with material that was available at home.

Activity 2: Weighing with their weighing scale

Class setting

The children worked individually and picked up any material in the playground and weighed it against other materials. Children were resourceful and used many objects for example, fresh leaves and dried leaves, stones, wet and dry sand, feathers and sticks. The children were asked to just play with the objects and the balance. The children naturally set out trying to find a balance, by putting weights and objects on the two different pans of the balance. The role of Diyas was to observe how they were doing it. Were they able to subtract the weight when it was more or did they keep on adding? Without interrupting, it was observed if they could work out when to add or remove weights. Did the children discover that adding weights on the opposite pan worked just like taking away weights from the first pan?

Activity 3: Introducing standard weights

Requirements

Different weights and balance

Class setting

Everyone sat in a circle. The Diya showed a weight of 1kg and two weights of 500gm. "Tell me which will be heavier? 1kg or two 500gm"

Some children answered, "Bhaiya, obviously the two 500gm!"

The Diya put the two 500gm weights in one pan and the 1kg weight on the other side.

Children: "Oh! Both are the same weight."

One child: "Yes bhaiya, those two 500 gm together becomes 1kg because 1kg is 1000gm"

The other children also realised that 1kg=1000gm.

To confirm, the Diya placed ten 100gm weights in one pan and 1kg in the other. The children observed the balance, added all the weights and concluded the same. To assimilate this, a question was put before them, "How many 200gm weights will weigh the same as 1kg.?" Some children calculated mentally. Some came forward and used the balance to find their answer. Finally all agreed that five 200gm becomes 1kg. In this way, we introduced weights like 10gm, 50gm, 100gm, 200gm, 1kg and 2 kg.

Activity 4: Weighing different objects with the help of weights

Requirements

- Balance(s) and weights
- Objects like bags, books, wooden blocks, bricks, water bottles and pencil box

Weighing school bag with weighing machine

Class setting: The children formed as many groups as there were balances and weights (at least two children per balance). They measured the weights of different objects and recorded these in a tabular form.

Name of the 'objects'	Weight
Notebook	
Bag	
Pencil box	
Crayon box	

The Diya checked when children were writing the weight. Were they adding correctly or not? Were they balancing properly? While writing, the children were guided to write the units, like kilogram or gram.

Activity 5: Guessing the weight of objects and checking estimates

Requirements: Small balance, objects like pencil, eraser, sharpener and pencil box.

Class setting: Each child had a separate balance.

Checking the accuracy of her estimation

Manipulating with quantities to bring balance

The children were asked to guess the weight of their own objects like pencil, eraser, sharpener and pencil box. They recorded these in their notebooks in a tabular form, found the correct weight using a balance, and checked the accuracy of their estimates.

Name of Object	Guess weight	Real weight
Pencil		
Eraser		
Pencil box		
Notebook		

Activity 6: Making a weight chart of the group

The children were asked to check their own weight with a weighing machine and write it on a piece of paper. They made a list of the children in the group with their respective weight. Using this, they identified the heaviest and lightest child in the group.

Activity 7: Worksheet on converting kilograms to grams and vice versa

1. Write in kilograms and grams

Example: 1250gm = 1kg and 250gm

- 1452gm =………. and……..
- 1820gm = ……….and ……..
- 2565gm = ………. and…….
- 4748gm = ……….and …….
- 7534gm =………..and……..

2. Write in grams

Example: 1kg and 276gm = 1276gm

- 1kg and 354gm = ...
- 4kg and 345gm = ...
- 3kg and 780gm = ...
- 8kg and 523gm = ...
- 6kg and 946gm = ...

Conclusion

This project, besides providing children with opportunities to use their thinking faculty, also encouraged them to use their power of estimation. The children worked extensively with their hands to bring about accuracy during the project. This brought quietness inside them. They worked individually as well with partners, which helped to develop coordination among friends, gave a sense of freedom and confidence to work individually and in a group. This, we believe, would pave the path for a harmonious journey into the future.

Project: Market

Progress Group, Age 8+

When the children of age group 8 were doing weight measurement, they went to a store to see how the items were packed, labelled and priced. Seeing that they got fascinated. So, we took the topic, Market. Through this project we tried to relate money with measurement of weight, volume and length.

Aims and objectives

- To develop the skill of weight measurement

- To learn the exchange of money
- To practise the skills of the four operations
- To learn about the barter system
- To gain experience of markets and shopping
- To learn the calculation of prices for different quantities

Activities

- Collecting and packing flowers, leaves, seeds and sticks
- Collecting materials like cloth, shoes, toys, books and sports materials
- Weighing sand and stones and making them into packets of different weights like 100g, 250gm, 500gm and 1kg
- Making some pictures and dolls out of paper and cotton
- Collecting toffees, sweets, or pieces of jaggery, individually packed (if not available, the children can make fake ones with mud)
- Making labels for each item with name of item, weight and price
- Labelling each item
- Setting up the market with stalls of each child
- Making paper money of different denominations
- Selling and buying items using paper money and real old coins (or handmade coins similar to the paper money)

Process

The children collected different materials from Nature like stones, sand, sticks, leaves, seeds and flowers. They also collected old materials like cloth, shoes, toys, utensils, books and sports materials from the school premises. The materials were packed after weighing them on a scale. They also made things like dolls and toffees out of paper and mud and packed them individually. The packets were labelled with names of vegetables, fruits, rice, dal, toffees and sweets. The weight of the contents and the cost price were also marked on the labels.

Paper money of different designs

Bankers at a meeting

While making labels for different items, the children were engrossed in calculating the price for different weight of different items. This was challenging for them because the operations on money were happening according to the measure.

After preparing everything they put their shop together as a market. Half of the children set up the shop as shopkeepers and the other half became customers. The shopkeepers

Exchange between shopkeeper and customer

made some rules for their shops. In the beginning, money was not introduced and the transactions were by barter. The rule was that one could buy anything by exchanging it with something else, in quantities that they decided.

Next, the customer and shopkeeper were given some money from the group bank. The paper money was used for this. The market opened. Customers came and shopped. After shopping, they had to calculate how much money was to be paid and how much money they needed to return to the bank. Both shopkeeper and customer were engaged in calculating. At the end of the market the shopkeeper needed to give the account to the Diyas and return money to the bank. Similarly, the customer had to give an account of the money spent in shopping and return the rest to the bank. The following day, they exchanged roles and the same activity took place.

Conclusion

This market continued for a week, and children from other groups came to the market. The children enjoyed taking part in all the activities related to the topic which gave them plenty of practice in addition, subtraction and multiplication. Their mental calculations got a boost in this project. Initially, when they went to shop and found that they did not have the exact money in change, they were not able to shop. Concerns were voiced, such as, “Bhaiya, I do not have 20-rupee notes. How can I buy?” This was articulated even by children who had with them 50 or 100-rupee notes. Gradually, they learnt the exchange of money, and how to pay more money and expect change from the shopkeeper. Those not able to calculate mentally were encouraged to use a notebook.

The shopping itself exposed the wonderful and amusing diversity of nature among the children. Some used to bargain. Others complained about high prices. One gave the money to the shopkeeper without counting, saying, "Take what you want." One little shopkeeper sold the goods, carefully wrote down the amount and gave it back to the costumer along with the goods!

Many children found it difficult to calculate the price for a particular weight of things. The Diya was at hand, helping to clarify things individually, when the market was in session and also, long after that when children were making sense of the idea of marketing and money.

APPENDIX

Curriculum

Mathematics Curriculum in mirambika

Red Group Age 3+

Initial Mathematics is introduced as a language and the children learn the language while using it in everyday life situations.

The conversation happens in Hindi language. Introducing vocabularies like मिलजुलकर, इकट्ठे, हम सब, हमारा, अपना-अपना, हर-एक, सीधा-उलटा, आमने-सामने, आए-गए, बड़ा-छोटा, लम्बा-नाटा, मोटा-पतला, पुराना-नया, हल्का-भारी, ज़्यादा/अधिक-कम, दूर-पास, पूरा-आधा।

Introduction of pre-mathematical concepts

- Matching, sorting and classification of things on the basis of shape, size, colours and materials is practiced regularly while the children put back the play materials and things of the group in respective places after playing.
- One to one correspondence with objects happens while keeping their bags, bottles, shoes and group materials in their respective places.
- Listening to numbers up to ten at different times in daily life situations.

Introduction of shapes and colours

Colours and shapes topic are taken to create an environment of experiential learning for children.

Blue Group Age 4+

Initial Mathematics is continued more as a language in Hindi and English through games, stories and situations in daily life. Children's faculty of intuition is nurtured.

- बड़ा, उससे बड़ा, सबसे बड़ा, छोटा, उससे छोटा, सबसे छोटा, सबसे लम्बा, सबसे मोटा, सबसे भारी, सबसे हल्का, उससे बड़ा, उससे छोटा
- Numbers are introduced in English

Pre-mathematical concepts are practiced more through games

- Matching and comparison (finding similarities and differences)
- Classification and sorting on the basis of materials, uses and textures
- Ordering and sorting on the basis of number, size, weight and shades of colour
- Pairing, one-to-one correspondence

Making relationships with shapes and size

- Recognising more shapes, observing shapes in everything and relating shapes with nature
- Making complex designs with blocks of different shapes and sizes

Exploration with 'pattern and design'

- Making pattern and design on the floor with natural and man-made things
- Observing patterns in nature, in games and in each other
- Repeating patterns and extending a given pattern

- Making story with patterns and designs
- Recognising the patterns in daily life

Relationship with numbers

- Able to count 20 objects with one-to-one correspondence
- Doing simple addition and subtraction through games and stories within ten objects

Green Group Age 5+

Further development of Initial Mathematics in Hindi and English language takes place.

Developing the number sense up to 100 and experiencing the four operations with objects

- Introducing 'Ganit mala'
- Counting objects in groups of ten
- Skip counting of 2, 3, 4, 5 and 10 with objects
- Breaking ten numbers into two or more parts
- Making bundles of tens and showing the exact number of sticks
- Carrying out four operations with ten to fifteen objects through story sums by first touching and then by looking at the objects

Developing the sense of estimation

- Playing games related to estimation of numbers, length, volume and quantity and weight

Making relationships with different shapes and sizes

- Developing the sense of fitting shapes like fitting a big triangle with small triangles
- Making more complex designs with blocks of different shapes and sizes
- Making rangolis on paper with colors as well as on floor with natural materials

Introducing Time as a topic

- Observing the sun's movement and shadow
- Experiencing the play of light and shadow
- Making sand clock, water clock and sun dial and using them
- Following a daily routine to understand the concept of time like day, hour, minute and second
- Learning clock reading skills

Introducing Measurements

- Playing measurement games related to length, weight, volume and capacity
- Measuring the length with hand span, cubit and foot span
- Making a toy balance and observing and comparing the heaviness and lightness of different objects
- Measuring with the balance and different sized containers

Yellow group Age 6+

Developing the mental faculties; sharpening and strengthening the skills

- Develop mental faculties like observation, concentration and reasoning
- Making the mind sharper and quicker
- Develop the ability to solve daily life situations through problem solving
- Develop visualization capacity to widen the mind

Developing the number sense and experiencing the four operations

- Counting till 1000 in English and till 100 in Hindi
- Skip counting by 2, 3, 4, 5, 6, 7 and 10
- Developing the ability to structure counting by using patterns of 10
- Grouping objects into hundreds, tens and ones
- Splitting the numbers into hundreds, tens and ones
- Identifying the position of number in 1000 bead-strings
- Making stick bundles of 100, 10 and 1
- Developing the sense of larger number and smaller number
- Adding and subtracting two and three-digit numbers with the use of sticks and beads
- Playing games related to addition and subtraction
- Making stories related to addition, subtraction, multiplication and division using two-digit numbers

Measurement related to money, length and time

Introducing money

- Playing with coins and paper money
- Making sense out of different denominations of coins and finding the relationship between different denominations of coins and notes

Length and width

- Measuring length and width with hand span, cubit, foot span, paper strip and drawing board

Time

- Continuing with the reading of clock and introducing small story sums related to time

Understanding the relationship between different shapes and sizes

- Identifying complex shapes
- Playing with rangometry and making of a bigger shape by using smaller shapes
- Making one shape using different shapes
- Making different 2-D and 3-D shapes with straw and building blocks

Introducing Abacus

- Calculating with code 10
- Making and identifying numbers in abacus
- Adding and subtracting using abacus

Orange Group Age 7+

Developing the mental faculties; sharpening and strengthening the skills

- Develop mental faculties like observation, concentration and reasoning
- Develop speed and accuracy in calculation
- Develop ability to solve daily life situations through problem solving
- Develop visualization capacity and widen the mind

Calculation with abacus

- Using code 2, 3, 5, and 7
- Solving word problem (addition and subtraction) with abacus

Going ahead with four operations

- Making relationship with 4-digit numbers
- Solving word problems through mental calculation with three-digit numbers
- Creating word problems, riddles, puzzles and games related to the four operations
- Skip counting with 4, 6, 7, 8, 9, 10, 11 and 12
- Splitting the numbers into 1000, 100, 10 and 1
- Developing the sense of place value
- Forming smallest and biggest number by using given digits
- Developing the ability of doubling and halving numbers
- Dealing with odd and even numbers
- Dealing with ascending and descending numbers

Introducing written arithmetic

- Carrying out four operations in the notebook
- Creating and solving word problems

Building a relationship with the shapes

- Playing with tangrams and making different designs
- Playing with tangrams and making triangles and squares by using 2, 3, 4, 5, 6 and 7 pieces

Measurement

Developing the power of estimation and measuring skill

- Measuring dimension using hand span, foot span, cubit, measuring stick and meter scale
- Measuring weight using balance and weights
- Measuring volume using different measuring glasses and bottles
- Taking up the topic on 'Calendar' through which the children build up an understanding of time in a wider sense while measuring time as hour(s), day(s), week(s), month(s) and year(s)
- Understanding 'money' by identifying with coins and notes while carrying out the four operations through the shopkeeper game

Bibliography

References in Chapter 2 are given below in an abbreviated form. The initial numeral is the serial number of the passage in this chapter located at the end of the selected passage. This is followed by the title of The Mother and Sri Aurobindo's writings, followed by volume number and page number.

CWM indicates Collected Works of the Mother

Essays in philosophy and yoga, Early cultural writings and The Renaissance in India are written by Sri Aurobindo.

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